ASME V&V 10.1-2012

An Illustration of the Concepts of Verification and Validation in Computational Solid Mechanics

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FOREWORD

From comments by the readership of the ASME Guide to Verification and Validation in Computational Solid Mechanics [1] (henceforth referred to as V&V 10), the ASME V&V 10 Committee on Verification and Validation in Computational Solid Mechanics recognized the need for another document that would provide a more detailed step-by-step description of a V&V application. The present document strives to fill that need by applying the general concepts of V&V to an illustrative example.

The authority of a standards document derives from the consensus achieved by the members of a standards committee (about 20 active members in V&V 10), whose interests span a broad range. Achieving such consensus is a long and difficult task, but the ultimate benefit to the computational mechanics community justifies the effort. Many compromises were made in the creation of the present illustrative example document. The main balance sought was to communicate to the reader on a basic level without distorting the many nuances associated with the exacting principles of verification and validation. The danger with being too basic is that the reader might take simplified concepts and statements out of the context of the illustrative example, and generalize them to situations not intended by the authors. The corresponding danger with being too exacting is that the reader might neither understand nor desire to understand the subtle points introduced by repeated qualification of terms (e.g., a "validated model" versus "a model validated for its intended use"). In most cases, the Committee favored clarity over completeness.

The scope of the document has evolved considerably since its inception. For example, the initial intent was to include as a lead-off example a one-experiment-to-one-calculation comparison without regard for uncertainties in either, since this is easiest to communicate and relate to readers and their possible past experience with validation. However, as a result of internal discussions and external reviews, the Committee came to accept that to maintain consistency with V&V 10, the recommended validation procedures must always account for uncertainties in both the calculations and the data that are compared. This led the Committee to restrict attention to validation requirements and metrics that depend directly on the underlying distributions characterizing the uncertainties in the calculations and data.

ASME V&V 10.1-2012 was approved by the V&V 10 Committee on December 2, 2011, the V&V Standards Committee on February 20, 2012, and the American National Standards Institute as an American National Standard on March 7, 2012.

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The Committee welcomes proposals for revisions to this Standard. Such proposals should be as specific as possible, citing the paragraph number(s), the proposed wording, and a detailed description of the reasons for the proposal, including any pertinent documentation.

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Subject: Cite the applicable paragraph number(s) and the topic of the inquiry.
Edition: Cite the applicable edition of the Standard for which the interpretation is being requested.
Question: Phrase the question as a request for an interpretation of a specific requirement suitable for general understanding and use, not as a request for an approval of a proprietary design or situation. The inquirer may also include any plans or drawings, that are necessary to explain the question; however, they should not contain proprietary names or information.

Requests that are not in this format may be rewritten in the appropriate format by the Committee prior to being answered, which may inadvertently change the intent of the original request.

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AN ILLUSTRATION OF THE CONCEPTS OF VERIFICATION AND VALIDATION IN COMPUTATIONAL SOLID MECHANICS

1 EXECUTIVE SUMMARY

This Standard describes a simple example of verification and validation (V&V) to illustrate some of the key concepts and procedures presented in V&V 10. The example is an elastic, tapered, cantilever, box beam under nonuniform static loading. The validation problem entails a uniform loading over half the length of the beam. The response of interest is the tip deflection. The validation test plan and the metrics and accuracy requirements for comparing the calculated responses with measurements are specified in the V&V Plan, which is developed in the first phase of the V&V program. In setting validation requirements and establishing a budget for the V&V program, the V&V Plan considers the level of risk in using the model for its intended purpose. Successfully meeting the V&V requirements means that the computational model for the tapered beam has been validated for the intended use discussed in this document, viz., predicting the response of a tapered beam tested in the laboratory.

To encompass as much of the general V&V process as possible in this example, a computational model was developed specifically for the tapered beam problem, even though it is more likely that a general-purpose finite-element code would be used in practice. The conceptual model is a Bernoulli–Euler beam for which the governing equations are solved with the finite-element method. The computational model was verified (checked for proper programming of the mathematical model and the solution procedure) by comparing computed values of tip displacement with an analytical solution to a relevant but simpler problem. A mesh refinement study initially revealed that the model did not converge at the expected theoretical rate. Further diagnosis revealed a programming error, correction of which led to the proper convergence rate. Knowing the allowable error due to lack of convergence then allowed an appropriate level of mesh refinement to be selected.

For validation (comparing with experimental results), 10 virtual trials of the same test were performed to quantify the distribution of results due to unintended variations in material properties, construction of the test specimens, and test execution. Other virtual tests were conducted to characterize uncertainties in selected model input parameters, namely rotational support stiffness and elastic modulus.

The following two validation approaches were considered:

(a) a case where uncertainty data were not available and obtained instead from subject matter experts.

(b) a case where uncertainty data were available from repeat tests and calculations.

In both cases the same metric was employed to demonstrate the use of uncertainty information in the model-test comparison. The validation metric is a measure of the relative error between the calculated and measured tip deflection of the beam.

The same model was used in both validation cases. In each case, the metric was compared to an accuracy requirement of 10%. In both cases the model was validated successfully. Had the validation been unsuccessful, it would have been necessary to correct any model deficiencies, collect additional or improved experimental data, or relax the validation requirement.

2 INTRODUCTION

This Standard is the first in a planned series of documents elaborating on the verification and validation topics addressed initially in the ASME V&V 10 Committee's seminal document, Guide to Verification and Validation in Computational Solid Mechanics (ASME V&V 10) [1]. V&V 10 was intentionally written as a high-level summary of the essential principles of verification and validation.

The present document provides a step-by-step illustration of the key concepts of verification and validation. It is intended as a primer that illustrates much of the methodology comprising verification and validation through a consistent example.

The example selected is a tapered cantilever beam under a distributed load. The deformation of the beam is modeled with traditional Bernoulli–Euler beam theory. The supported end of the beam is fixed against deflection but constrained by a rotational spring. This nonideal boundary condition, along with variation in the beam's elastic modulus, enables us to illustrate the treatment of uncertain model parameters.

The illustrative portion of the document begins with the Verification and Validation Plan (section 5). This plan is the recommended starting point for all verification and validation activities. The V&V Plan provides the

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framework for conducting the verification and validation assessment and provides an outline for the timing of activities and estimating required resources. The V&V Plan is developed as a team effort with participation by the customer (who provides the requirements), decision makers, experimentalists, modelers, and those who will perform the validation comparisons.

Having agreed on a V&V Plan, the next step is model development, which includes three types of models: conceptual, mathematical, and computational (section 6).

Developed on a parallel path with the mathematical and computational models are the validation experiments — the physical realizations of the reality of interest (tapered cantilever beam) that will eventually serve as the referent against which the computational model predictions are compared in the validation phase.

Once the computational model has been developed, an assessment of the agreement between the statement of the mathematical model and the results from the computational model is required. This activity is called verification and comprises two main parts: code and calculation verification (section 7). Code verification is typically performed by comparing the results from analytical solutions to the corresponding computational model results. Calculation verification is performed with successive grid refinements and estimations of the discretization error using techniques based on Richardson extrapolation [5].

Having verified that the computational model is mistake-free for the cases tested, and having established a level of mesh refinement that produces an acceptable discretization error, the predictive calculation of the validation experiments can proceed. In parallel, the validation experiments can be conducted and results recorded. The validation assessment is then made by comparing the outcomes of the model prediction and the validation experiment to determine whether the validation requirement has been satisfied.

The remainder of this document is organized as follows. The Purpose and Scope (section 3) describes which parts of the V&V process are, and are not, covered by the illustrative example. Next, the Background (section 4) provides a review of the verification and validation process and describes how the illustrative example fits into an overall validation hierarchy — a key element in the validation process. Sections 5 through 9 contain the illustrative example. The document ends with a brief Summary (section 10), which restates the key results from the illustrative example, and finally some Concluding Remarks (section 11) providing a look to the future of ASME V&V 10 verification and validation activities.

3 PURPOSE AND SCOPE

The purpose of this document is to illustrate, by detailed example, the most important aspects of V&V described in the Committee's framework document, Guide to Verification and Validation in Computational Solid Mechanics (V&V 10). V&V 10 intentionally omitted examples, as its purpose was to provide "a common language, a conceptual framework, and general guidance for implementing the process of computational model V&V," an already broad scope for a 27-page consensus document. The present document is the first in a series of more detailed and practical ones the Committee has planned to incrementally fill the gap between V&V 10 and a set of recommended practices.

To appeal to a broad range of mechanics backgrounds, a cantilever beam problem has been selected to illustrate the following aspects of V&V. The numbers in parentheses are references to sections and paragraphs in V&V 10 as follows:

(a) validation plan (para. 2.6)
  (1) validation testing (para. 2.6.1)
  (2) selection of response features (para. 2.6.2)
  (3) accuracy requirements (para. 2.6.3)

(b) modeling development (section 3)
  (1) conceptual model including intended use (para. 3.1)
  (2) mathematical model (para. 3.2)
  (3) computational model (para. 3.3)

(c) verification (section 4)
  (1) code verification: comparisons using analytical solution (para. 4.1)
  (2) calculation verification: mesh convergence (para. 4.2)

(d) parameter estimation (para. 3.4.1)

(e) validation (section 5)
  (1) validation experiments (paras. 5.1 and 5.2)
  (2) comparison of experimental results and model prediction (para. 5.3)
  (3) decision of model adequacy (para. 5.3.2)

(f) uncertainty quantification (para. 5.2)

(g) documentation (paras. 2.7, 4.3, and 5.4)

4 BACKGROUND

V&V is required to provide confidence that the results from computational models used to solve complex problems are sufficiently accurate and indeed solve the intended problem. The conceptual aspects of V&V are described in detail in V&V 10. V&V is used with increasing frequency in recognition that confidence in ever increasingly complex simulations can only be established through a formal, standardized process.

V&V includes assessment activities that are performed in the process of creating and applying computational models to address technical questions about the performance of physical systems. The overall process is summarized in Fig. 1, which is taken from V&V 10. The present document describes and provides examples of

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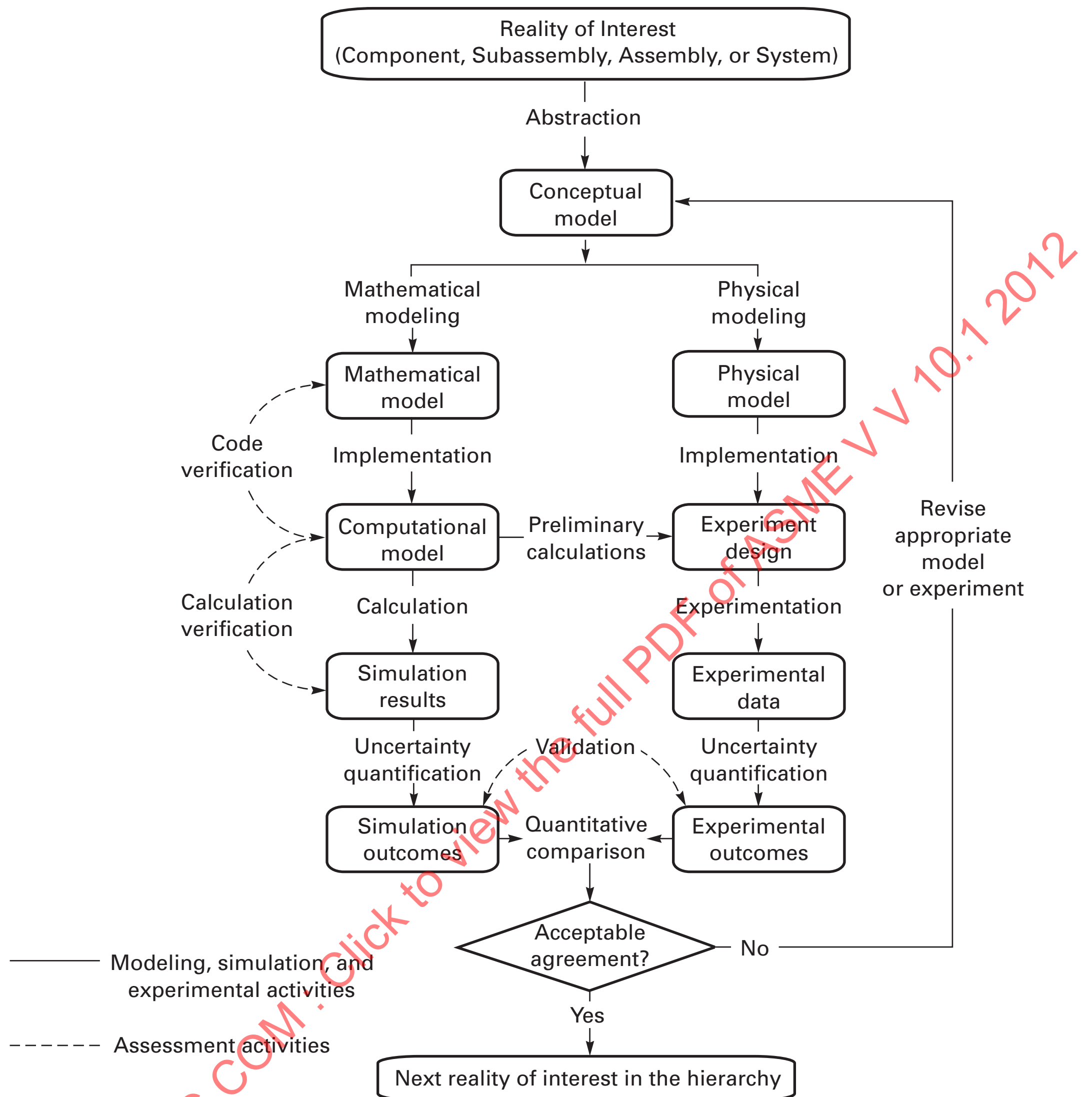
Fig. 1 V&V Activities and Products



GENERAL NOTE: This figure also appears as Fig. 4 in V&V 10 [1].

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most activities in the figure in a presentation unencumbered by the complexities of a typical real-world physical system and its associated mathematical model.

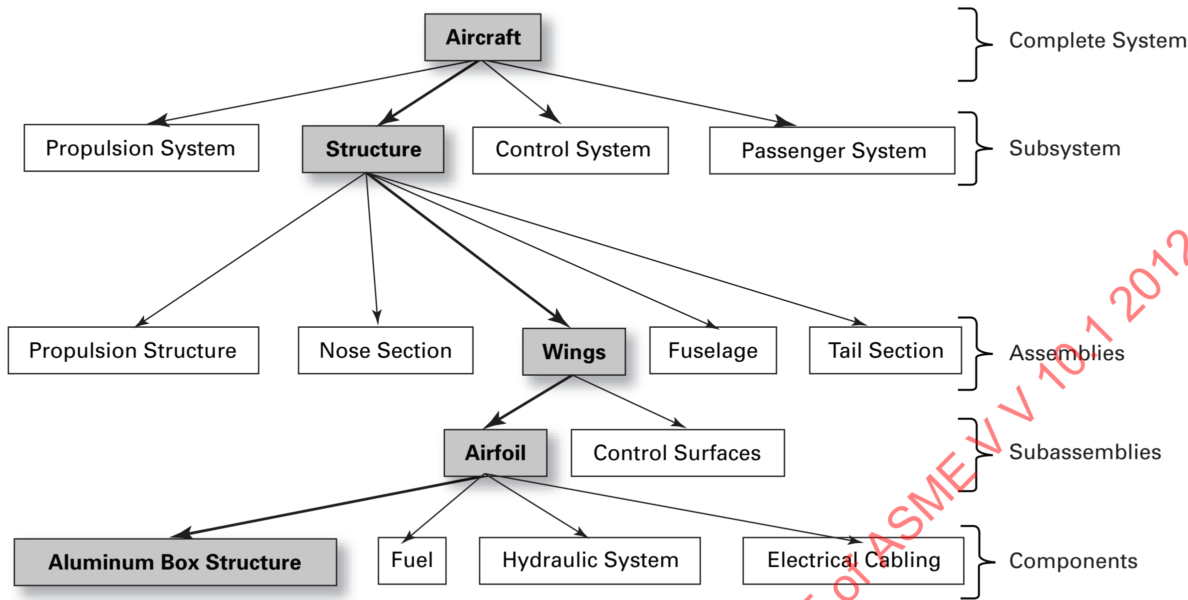
4.1 Verification

Verification is defined as the process of determining that a computational model accurately represents the underlying mathematical equations and their solution. Verification has two aspects: *code* verification and *calculation* verification. Code verification is defined as the process of ensuring that there are no programming errors and that the numerical algorithms for solving the discrete equations yield accurate solutions with respect to the true solutions of the governing equations. In addition to *numerical* code verification — illustrated here via a convergence analysis — code verification also includes formal software quality engineering (SQE) processes; SQE will not be discussed in this Standard. Calculation verification is defined as the process of determining the solution accuracy of a particular calculation. Both numerical code verification and calculation verification will be demonstrated by applying a simple beam element code to successively more finely meshed models of statically loaded beams.

4.2 Validation

Validation is defined as the process of determining the degree to which a computational model is an accurate representation of the real world from the perspective of the intended uses of the model. One of the steps in the validation process is a comparison of the results predicted by the model with corresponding quantities

Fig. 2 Validation Hierarchy Illustration for an Aircraft Wing



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observed in the validation experiments, so that an assessment of the model accuracy is obtained. Several possible validation activities will be mentioned with regard to the simple beam example.

An important initial step in the model development process is to define what is to be predicted and how accurate the prediction needs to be. These will in turn depend on system design requirements and on possible consequences of system or subsystem failure, as well as project cost and schedule requirements. The validation example used in this document is driven by customer requirements for the accurate prediction of aircraft wing tip deflection. For the example, however, we shall only address validation of a computational model for a simplified laboratory structure that is related to an aircraft wing.

Having defined the top-level reality of interest as the wing of a complete aircraft, the model development team can proceed to construct a validation hierarchy. An example of such a hierarchy is shown in Fig. 2. The validation hierarchy starts as a top-down decomposition of the physical system into its subsystems, assemblies, subassemblies, and components. From the bottom up, the hierarchy can be seen as a sequence of specific and relevant models representing distinct realities of interest leading to the one ultimately to be modeled.

Careful construction of a validation hierarchy is of paramount importance, because it defines the various problem characteristics to be captured by different elements in the hierarchy, it encapsulates the coupling and interactions among the various elements in the hierarchy, and it suggests the validation experiments that should be performed for each element. Each element in the hierarchy requires a model, the validation of which provides an estimate of its degree of accuracy and evidence of its adequacy for use in the complete system model. The component model labeled "Aluminum Box Structure" in Fig. 2 represents the tapered cantilever beam discussed from this point forward. The cantilever beam model represents one particular reality of interest in the overall aircraft wing validation program.

If experimental data cannot, or will not, be obtained at some higher tier, then model accuracy assessment cannot be accomplished at that tier. As a result, a prediction must be made for the higher tier using the general concept of extrapolation of the model for the conditions of the intended use at the higher tier. For this situation, an estimate of the total uncertainty in the prediction must be made for the system response quantities of interest. The total uncertainty at the higher tier is a combination of

(a) the uncertainty in the model and the experimental data at the tier where experimental data are available

(b) the uncertainty in the modeling of the coupling among the models occurring at the higher tier

(c) the uncertainty in the extrapolation to the higher tier of the model along with all of the input data to the model

4.3 Documentation

When a V&V effort ends, the most definitive statement of that effort that will endure is the V&V documentation. Considering the significant resources typically required for any V&V effort, to leave the effort undocumented, or poorly documented, squanders the future benefit of those expended resources.

Just as the V&V process goes beyond simply conducting calculations in its attempt to establish the credibility of the results, so too the documentation of the V&V process needs to go beyond merely reporting quantitative results. Therefore, it is not sufficient for V&V documentation to provide only physics equations and tables of numbers that summarize a set of complex simulations. Rather, the essence of V&V documentation is to provide the rationale for the selected physics equations, list assumptions, define metrics, explain the relationship between numerical and experimental results, and catalog uncertainties. For example, an explanation of the rationale for the choice of a particular constitutive model will have more future value than a mere listing of the constitutive model parameters. The V&V documentation reader will want to know the answers to "why" questions as much as the quantitative results.

The scope and level of detail of the V&V documentation will depend on the potential consequences of the intended uses of the model, with routine in-house projects probably requiring fairly brief and straightforward documentation. At the other end of the spectrum, much more detailed documentation will be required for high-consequence model predictions or, for example, when results will be submitted to regulatory agencies for approval to proceed with new designs or products.

To be done properly, V&V documentation should start at the beginning of the effort by documenting the V&V Plan (section 5), and then proceed through all phases of the V&V effort, recording not only the successes but also the failures; failures often teach more than successes.

V&V documentation should comply with available local and formal guidance related to the intended model use and area of application. For example, MIL-STD-3022 [2] has templates for the V&V Plan and V&V Report for simulations within Department of Defense activities.

5 VERIFICATION AND VALIDATION PLAN

The V&V Plan is a document that describes how the V&V process will be conducted. It is prepared after the validation hierarchy has been defined but before many of the details of the model or experiments have been fleshed out. It is primarily driven by customer requirements, but should reflect the views of all interested parties — decision makers, analysts, and experimentalists — regarding what the modeling effort is to accomplish and — to a limited extent — how it is to be accomplished.

The starting point for a V&V Plan is the customer's description of the top-level reality of interest and of the intended use of the model. Next, with a level of customer involvement that depends on the customer's modeling or testing experience, a validation hierarchy should be developed as exemplified in Fig. 2. The key items in the V&V Plan are

(a) description of the top-level reality of interest

(b) statement of the intended uses of the top-level model

(c) validation hierarchy, including descriptions of all lower level realities of interest and intended uses

(d) for each item in the hierarchy, the following:

(1) selection of response features [i.e., system response quantities (SRQs)] to be measured, computed, and compared

(2) statement of verification requirements, including (as applicable) software engineering methods and iterative or spatial convergence checks

(3) metrics to be used for comparing computed results with experimental measurements

(4) model accuracy requirements (i.e., ranges of the metrics for which the simulation will be deemed adequate for its intended use)

(5) recommended courses of action if model accuracy requirements are not satisfied

(6) a list of validation experiments to be performed

(7) estimated cost and schedule

(8) programmatic assumptions and limitations (e.g., assumptions about being able to conduct particular types of experiments, or decisions to use previously validated submodels or available experimental data)

All of these items except the last two will be briefly discussed in the next few subsections in the context of the selected example. Further detail about SRQs, validation metrics, and validation accuracy requirements will be provided in sections 8 and 9.

5.1 Reality of Interest, Intended Use, and Response Features

Referring to Fig. 2, an assemblies-level reality of interest is an aircraft wing. However, the Committee's collective experience has shown that failure to validate models at lower levels in the validation hierarchy is often the cause for subsequent model validation failures at higher levels in the hierarchy. Thus, among the various lowest level components shown in Fig. 2, the decision was made to validate the response of the "Aluminum Box Structure." This is the present illustration's reality of interest. Specifically, the structure to be considered, shown schematically in Fig. 3, is a hollow, tapered, cantilever beam under static loading in a laboratory environment.

The intended use of the model, in the context of this simple example, is to demonstrate competence in modeling a low-level component. If such low-level component models cannot be successfully validated, there is no hope of moving up the hierarchy to validate more complex subassemblies.

The SRQ of interest is the transverse tip deflection of the beam. In general, it is recommended to consider

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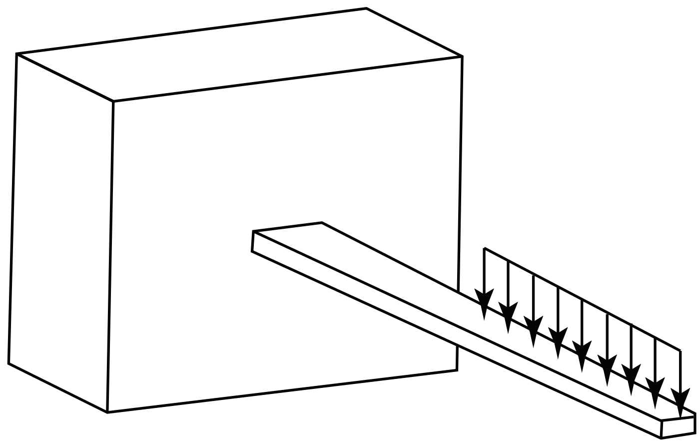
Fig. 3 Schematic of the Hollow Tapered Cantilever Beam



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multiple SRQs to enhance credibility in the model as a whole; however, for conciseness only the tip deflection is considered here.

For the planned experiments, tapered beams are to be embedded at their wide end into a stiff fixture approximating a "fixed-end" or cantilevered boundary condition. The beams are to be loaded continuously along the outer half of their lengths.

During the experiment planning, it was acknowledged that the "fixed-end" boundary condition can only be approximated in the laboratory. Thus in the model development the translational constraint at the boundary will be assumed fixed, but the rotational constraint will be assumed to vary linearly with the magnitude of the moment reaction.

Additionally, the beam model to be developed is assumed to have negligible shear deformation, and thus shear deformation is ignored in the mathematical model. For the prescribed magnitude of the loading, the deflection of the beam will be small relative to beam depth, so a small-displacement theory will be used, and the beam material is assumed to be linear elastic.

These assumptions feed directly into the conceptual model of the physical structure, which will be defined precisely in para. 6.1, and which guides both the development of the validation experiments and the definition of the mathematical model.

5.2 Verification Requirements

In this example, both code and calculation verification will be performed. The requirements for code verification are as follows:

(a) It is conducted using the same system response quantities as will be measured and used for validation.

(b) It demonstrates that the numerical algorithm converges to the correct solution of a problem closely related to the reality of interest as the grid is refined. This can be difficult or impractical in many cases, but without it, the code is not verified.

(c) It demonstrates that the algorithm converges at the expected rate.

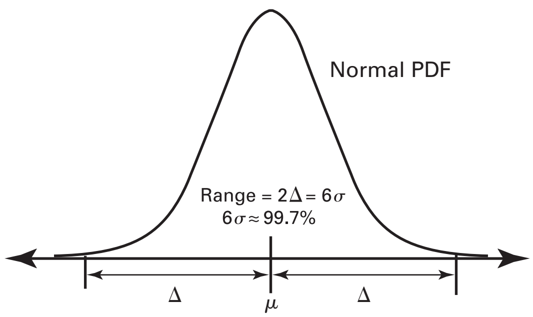
The requirement for calculation verification in general is to demonstrate that the numerical error (due to incomplete spatial or iterative convergence) in the SRQs of interest be a small fraction of the validation requirement. In this example, the validation requirement will be 10%, and the numerical error is required to be no greater than 2% of that (i.e., 0.2%).

5.3 Validation Approaches, Metrics, and Requirements

Two different validation *approaches* are demonstrated in this Standard. They differ mainly in the source of information used to quantify the uncertainties in the computed and measured values of the SRQ. The V&V Plan must specify which approach is to be taken. The plan must also specify a metric that incorporates the uncertainties in a way that provides a single measure of the relative difference between the simulation outcomes and the validation experiments. It must also specify a validation requirement, representing the maximum acceptable difference between simulation and experiment in terms of the selected metric. Measured results from a validation experiment form the referent against which the computational model predictions are compared (via the metric) to assess the accuracy of the computational model. After the experiments and calculations are completed, the metric is calculated and compared to the requirement to determine whether the model has been validated.

The metric that will be used is defined in such a way that the validation requirement may loosely be regarded as a maximum acceptable percentage difference between calculation and experiment. Validation requirements should be developed based on the needs and goals of the

Fig. 4 Estimating a Probability Density Function From an Uncertainty Estimate, Δ



application of interest. Some considerations in setting validation requirements are

(a) predictive accuracy of the computational model based on the requirements of the customer

(b) limitations based on obtaining experimental measurements because of diagnostic sensor capabilities, project schedule, experimental facilities, and financial resources

(c) the current point in the engineering development cycle from conceptual design to final design

(d) consequences of the engineering system not meeting performance, reliability, and safety requirements

Validation Approach 1 is a possible approach to take when only a single experiment and a single simulation are available. In this case, the uncertainties associated with the single values of the SRQ must be estimated. Specifically, subject matter experts must be identified for both the experiments and modeling. They may or may not be the experimentalist and modeler in the current assessment. Based on their past experience in related work, each is asked to estimate a symmetric interval within which all practical results are expected to lie. The half-width, Δ, of the interval is shown in Fig. 4. For convenience and simplicity in the absence of data, Δ is taken as the basis for constructing a Gaussian (normal) distribution of the uncertainty. In Validation Approach 1, the measured or calculated value of the SRQ is assumed to be the mean (μ) of the distribution, and the estimated half-width, Δ, is interpreted to be equal to three standard deviations (3σ). As shown in Fig. 4, the total range provided by Δ covers 99.7% of the probability. The standard deviation, σ, is easily computed from the given Δ.

Validation Approach 2 is similar to Approach 1, except the uncertainty in the experimental outcome is quantified through replicate tests,[1] and the uncertainty in the simulation outcome is quantified through a probabilistic analysis with uncertain model inputs, which are derived from different types of replicate tests. The resulting probability density functions (PDFs) therefore are not necessarily Gaussian or even symmetric.

The two validation approaches are notionally illustrated in Fig. 5.

In this example, the metric employed in either validation approach is based on the area between the measured and calculated SRQs' cumulative distribution functions (CDFs; the CDF is the integral of the PDF). This metric, sometimes referred to as the "area" metric, is illustrated in Fig. 6, and more detail about it is given below.

The area metric M^{SRQ} is the area between the experiment and model CDF [3], normalized by the absolute mean of the experimental outcomes. Thus, if $F_{SRQ}(y)$ is the CDF of either the model-predicted or measured SRQ values, then

$$M^{SRQ} = \frac{1}{\left|\overline{SRQ}^{exp}\right|} \int_{-\infty}^{\infty} \left| F_{SRQ^{mod}}(y) - F_{SRQ^{exp}}(y) \right| dy \qquad (1)$$

where

$\overline{SRQ}^{exp}$ = the mean of the experimental outcomes

This metric is nonnegative and vanishes only if the two CDFs are identical. To help understand what the metric represents, it can be shown that in the special case where the two CDFs do not cross, the integral in eq. (1) is the absolute value of the difference between the means, and in general, it is a lower bound on the mean of the absolute value of the difference between SRQ^{mod} and SRQ^{exp} [4]. In the deterministic case, where both CDFs are step functions, the area is simply the absolute value of the difference between the two unique values.

For both validation approaches in this example, the validation requirement is taken as

$$M^{SRQ} \le 0.1 \qquad (2)$$

Obviously, satisfaction of a particular validation requirement is the desired outcome of the validation assessment. However, the V&V Plan should include a recommended course of action if the validation requirement is not met. Such contingency plans could include improvements in the model, improvements in the validation experiments, better quantification of uncertainties, relaxation of the requirements, or some combination of these. The choice of which course to pursue depends on the application of interest, the consequences of failure to validate, and available resources.

[1] In constructing and analyzing this illustrative example, no physical experiments were performed. Throughout this document, any references to a specific test article, measurement, experiment, or experimental result, is to an item that has only been conjured for illustrative purposes.

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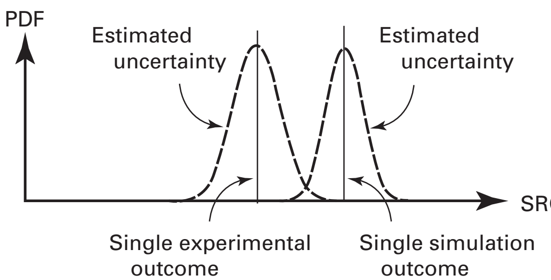
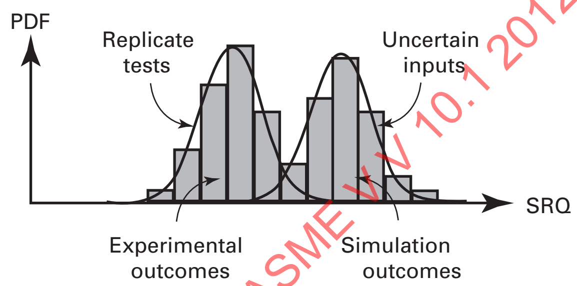
Fig. 5 Illustration of the Two Validation Approaches



(a) Validation Approach 1



(b) Validation Approach 2

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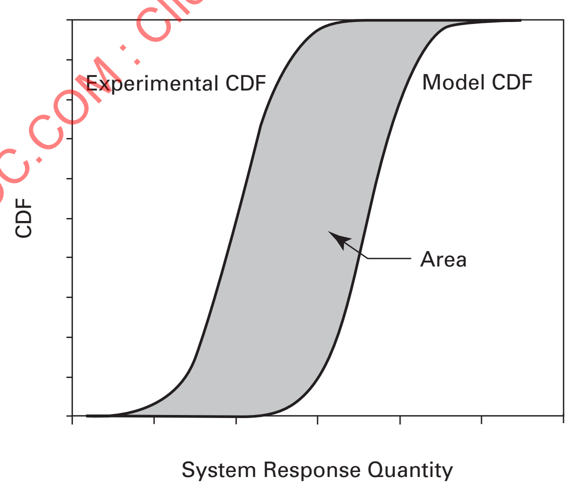
Fig. 6 Illustration of the Basis of the Area Metric



The metric we have defined is normalized by the mean of the experimental outcomes. This is in contrast with some other validation assessment procedures that suggest using a metric normalized by the sample standard deviation of the measured SRQ, or equivalently, requiring the absolute error to be within some specified number of standard deviations of the measurements. This is not recommended because it confounds two unrelated issues:

–the uncertainty in the experimental measurements

–the predictive accuracy required of the model for the application of interest

For example, one could have large experimental uncertainty making it easy for the model result to fall within two standard deviations of the measurements. Stated differently, model accuracy requirements should be *independently* set on how well the model reproduces features of the experimentally measured response. These requirements are determined by such things as engineering design and system performance requirements and not by sources of measurement uncertainty or the impact of input parameter uncertainty on system responses.

6 MODEL DEVELOPMENT

This section discusses the application of general modeling concepts to different phases of model development, with emphasis on the lowest level of the hierarchy, which is where the present example resides.

6.1 Conceptual Model

The conceptual model is the initial product of the process of abstracting the reality of interest. It includes the set of assumptions that permits construction of both the mathematical model and definition of the validation requirements. As indicated in Fig. 1, a conceptual model is required for each element of the hierarchy shown in Fig. 2. The conceptual model should be developed with a clear view of the reality of interest, the intended use of the model, the response features of interest, and the validation accuracy requirements. All of these items are defined in the V&V Plan. The conceptual model should consider physical processes, geometric characteristics, influence of the surroundings (e.g., loading), and uncertainties in each of these.

The example problem at hand concerns quasistatic deformation of a built-in, tapered, statically loaded, elastic beam. When the beam deforms, plane sections are assumed to remain plane and normal to the middle surface of the beam (i.e., Bernoulli–Euler beam theory applies). The beam structure and loading are assumed to be symmetric such that there is no twisting deformation. Shear deformation is neglected. Material properties are assumed to be isotropic, homogeneous, and linear elastic. Beam deflections are assumed to be small enough that geometric nonlinearity can be neglected.

Although physical realizations of any system will vary randomly, variation of most of the system-characterizing quantities in this illustrative example are considered to be small and will be neglected. The *two* exceptions in this example are the rotational stiffness at the wall and the modulus of elasticity of the aluminum. The variability in these two parameters cannot be eliminated or tightly controlled in the validation experiments, so in Validation Approach 2 the approach taken is to measure the variability (from separate characterization tests) and then include this variability in the probabilistic model. It is furthermore assumed that these two parameters are independent of each other. The foregoing assumptions about the conceptual model provide the basis on which the mathematical model is constructed. That none of them will be precisely satisfied in the validation experiment, and that many will vary from one repetition of the experiment to another, suggests why the model predictions, especially from a deterministic model, may not exactly match experimentally measured results.

In summary, the major assumptions to be used in development of the beam model are as follows:

(a) The beam material is homogeneous, isotropic, and linear elastic.

(b) The beam undergoes only small deflections.

(c) Beam deflections are governed by static Bernoulli–Euler beam theory.

(d) The beam and its boundary conditions are perfectly symmetric from side to side, and all loads are applied symmetrically; therefore, beam deflections occur in a plane.

(e) The beam boundary constraint is fixed in translation and constrained against rotation by a linear rotational spring.

6.2 Mathematical Model

The mathematical model uses the information from the conceptual model, including idealizing assumptions concerning the behavior of the beam, to derive equations governing the structure's behavior. For the beam considered here, the assumptions listed when defining the conceptual model in para. 6.1 combine to yield the equations of static Bernoulli–Euler beam theory:

$$
\begin{aligned}
&\frac{d^2}{dx^2}\left(EI(x)\frac{d^2}{dx^2}w(x)\right) = q(x),\ 0 \le x \le L,\\
&w(0) = \left.\frac{dw}{dx}\right|_{x=0} = f_r EI(0)\left.\frac{d^2w}{dx^2}\right|_{x=0},\ \left.\left[EI(x)\frac{d^2}{dx^2}w(x)\right]\right|_{x=L} = 0,\\
&\left.\frac{d}{dx}\left[EI(x)\frac{d^2}{dx^2}w(x)\right]\right|_{x=L} = 0,\\
&I(x) = \frac{1}{12}\left\{b_0\left(1-\alpha\frac{x}{L}\right)h^3 - \left[b_0\left(1-\alpha\frac{x}{L}\right)-2t\right]\left[h-2t\right]^3\right\}
\end{aligned}
\tag{3}
$$

where

b_0 = width at the support

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E = modulus of elasticity of the beam material
f_r = flexibility of the linear rotational spring restraining the beam at its constrained end
h = depth of the beam
$I(x)$ = area moment of inertia of the beam
L = length of the beam
$q(x)$ = distributed load in the y-direction
t = wall thickness
$w(x)$ = beam deflection in the direction normal to the undeformed centerline
x = measured from the supported end
α = taper factor 1-(tip width)/(base width)

The boundary conditions at the supported end represent the assumption of zero translation and a linear rotational spring constraint. At the free end of the beam there are the conditions of zero moment and shear. Under some circumstances eq. (3) can be easily solved in closed form, but when the area moment of inertia varies along the length, a numerical solution is usually sought.

The development of mathematical models in present-day computational solid mechanics typically does not include explicitly writing the differential equations. Rather, the computational model is constructed directly from the conceptual model via selection of element types, material models, boundary conditions, and associated options. This is not to say the differential equations are omitted from the modeling process; rather, the differential equations are documented in the software's user or theory manual (e.g., in descriptions of various beam element types and options). In that case, it is highly recommended that the analyst review those models, equations, and assumptions (given the options chosen in the code) to ensure they are consistent with the intended use of the model. Without carefully considering these equations, an error or inconsistency in the mathematical modeling can easily occur.

6.3 Computational Model

The computational model provides the numerical solution of the mathematical model, and normally does so in the framework of a computer program. The range of discretization approaches (e.g., finite element, finite difference) and options within each approach in commercial software is often extensive. The analyst needs to find a balance between representing the physics required by the conceptual model and the computational resources required by the resulting computational model. For example, finite element type options for the mathematical/computational model for the airfoil aluminum skin would include the following:

(a) continuum elements: use solid elements through the thickness of the aluminum skin

(b) shell elements: plane stress assumption through the skin thickness

(c) plate elements: same as shell element but omit surface curvatures

(d) hollow-section beam elements: strains are assumed to be primarily axial and torsional

(e) closed-section beam elements: equivalent constant cross-sectional properties of a solid section

(f) uniform closed-section beam elements: average cross section properties along length

In this example, the computer code used to solve for the beam deflections and rotations was specially written for this application example. It can be used only to analyze beam structures. It is a finite element program employing Bernoulli–Euler beam elements with constant cross section. If the element itself is loaded by any combination of uniform distributed load, transverse end forces, and couples at the ends, then the relative displacements and rotations at the ends are exact in the context of Bernoulli–Euler beam theory. On the other hand, when used to model a tapered beam, these relative deformations are only approximations.

The beam considered here is shown schematically in Fig. 3. The length of the beam is 2 m, the depth is 0.05 m, the width varies linearly from 0.20 m at the supported end to 0.10 m at the free end, and the wall thickness is 0.005 m. The material is aluminum, with a modulus of elasticity of 69.1 GPa. A uniform distributed load of 500 N·m is applied vertically in the downward direction on the outer half of the beam.

7 VERIFICATION

Code verification seeks to ensure that there are no programming errors and that the code yields the accuracy expected of the numerical algorithms used to approximate the solutions of the underlying differential equations. This is in contrast with *calculation* verification, which is concerned with estimating the discretization error in the numerical solution of the specific problem of interest. The distinction is subtle but important, because code verification requires an independent, highly accurate reference solution and can (and usually will) operate on a problem that is different from the problem of interest.

What is important in code verification is that all portions of the code relevant to the problem at hand be fully exercised to ensure that they are mistake free. This is done by comparing numerical results with analytical solutions, and in the process, confirming that the numerical solution converges to the exact one at the expected rate as the mesh is refined.

At the root of both code and calculation verification is the concept of the *order of accuracy* of a numerical algorithm. Under h-refinement (variation of element size, as contrasted with p-refinement or variation of algebraic order of interpolation functions), it is defined as the exponent p in the power series expansion

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$$w_{\text{exact}} = w_h + Ah^p + \text{H.O.T. as } h \to 0 \tag{4}$$

where

- A = constant
- H.O.T. = higher order terms, which tend to zero faster than the lowest order error term (the second term on the right) as h tends to zero
- h = grid size
- w_{exact} = exact solution
- w_h = numerical solution at grid size h

In the manipulations to follow, grid size is assumed to be small enough that the H.O.T. are negligible compared to the lowest order error term [5]. When this is so, the numerical solution is said to be in the *asymptotic convergence regime*.

Observed order of accuracy is the value of p inferred from eq. (4) when it is applied as detailed below to two or three numerical solutions at different grid resolutions. *Theoretical* order of accuracy is the value of p derived from a mathematical analysis of the algorithm. According to eq. (4), the slope of a log–log plot of the absolute value of numerical error $w_h - w_{\text{exact}}$ vs. grid size will tend to p as grid size tends to zero. Since in a linear array grid size is inversely proportional to number of elements, the slope of a plot of error vs. number of elements will tend to $-p$ as number of elements becomes large, as will be illustrated in para. 7.1.

7.1 Code Verification

There happens to exist an analytical solution to the main problem of interest, viz., a tapered cantilever loaded over half its length. While it would be tempting to use it for both code and calculation verification, we shall do neither, because in general such a solution will not be available. (If it were, there would be no reason to undertake a numerical solution in the first place.) Therefore in this example, numerical code verification will be performed on a slightly different problem, viz., the same tapered cantilever beam but with a uniform load *along its entire length*.

The key points here are as follows:

(a) While different, this problem is closely related to the problem of interest, as in general it must be to serve as the reference solution for code verification.

(b) The problem has an exact analytic solution.

Tip deflections at various grid refinements will be compared to that analytical solution, leading to estimates of the observed order of accuracy based on eq. (4). This will then be compared to the theoretical order of accuracy.

The analytic solution of a linearly tapered, uniformly loaded cantilever beam has been derived by integration of eq. $(3)_1$ with q = constant. The resulting normalized transverse deflection for a beam that tapers linearly in width is

$$\frac{EI_0}{qL^4}\,w(x) = \frac{1}{2\alpha}\left\{\left(\frac{1}{\alpha}-1\right)^2\left[\frac{x}{L}+\left(\frac{1}{\alpha}-\frac{x}{L}\right)\ln\left(1-\alpha\frac{x}{L}\right)\right] - \frac{1}{6}\left(\frac{x}{L}\right)^3 + \left(1-\frac{1}{2\alpha}\right)\left(\frac{x}{L}\right)^2\right\} \tag{5}$$

The taper factor α for the problem at hand is 0.5, so from eq. (5) the exact normalized tip deflection is $\frac{5}{6} - \ln(2) \approx 0.14018615$, which will be the basis for code verification comparisons.

Table 1 Normalized Deflections

Number of Elements	EI_0w/qL^4 Initial Coding	Final
2	0.13281250	0.14642858
4	0.13549805	0.14172980
8	0.13769015	0.14057124
16	0.13890418	0.14028238
32	0.13953705	0.14021021
64	0.13985962	0.14019217
128	0.14002239	0.14018766

Table 1 gives the numerically derived normalized tip deflections from two different sets of calculations. Figure 7 shows absolute values of the differences between these and the exact value. Based on algorithms similar to the one used here, the theoretical order of accuracy is expected to be 2; further, it can be shown analytically that the theoretical order of accuracy of the numerical algorithm when applied to this specific problem is in fact 2. First consider the results of the initial coding. The error plot strongly implies that the results are systematically converging to the exact solution, but even without performing further calculations, the slope of the line indicates an order of accuracy much closer to 1 than 2. During the early stages of the development of this example, these were the computed results. The unexpectedly low observed order of accuracy prompted a detailed review of the coding, and an error was found. Upon correction of that error, the second set of results was obtained. Now the observed order of accuracy as indicated by the slope of the error plot is seen to be much closer to the theoretical value of 2. The error was unintentional, and could not possibly have provided a better illustration of the value of numerical code verification.

It now remains to extract a precise numerical estimate for the observed order of accuracy of the corrected numerical solution. This is accomplished by computing the logarithmic slope between the last two points on the

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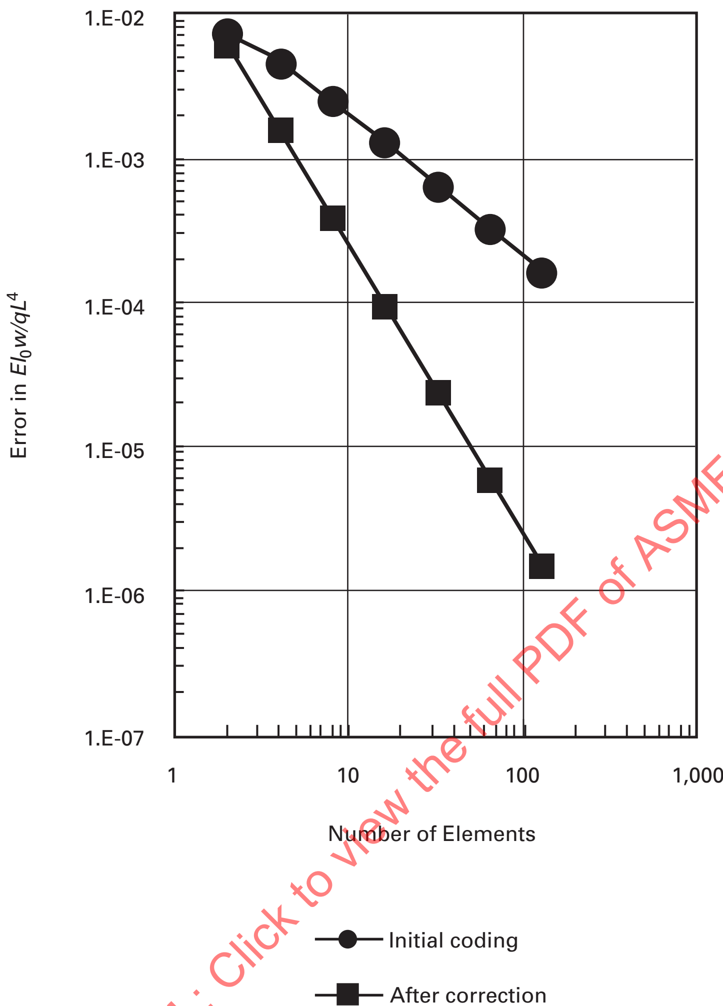
Fig. 7 Errors in Normalized Deflections



GENERAL NOTE: Note log–log scale.

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lower line in Fig. 7. Using the numbers to 8 significant figures as shown in Table 1, this comes out to −1.995.

When using commercial or other "black box" software, the theoretical order of accuracy p is typically not known. In such a case, the user can compare the software solution to an appropriate analytical solution and perform mesh convergence studies to derive the observed order of accuracy, exactly as was done in the previous paragraphs.

7.2 Calculation Verification

The goal of calculation verification in the present example is to estimate the numerical error in tip deflection as a function of discretization. In conjunction with an independently specified numerical accuracy requirement, this can then be used to guide the choice of discretization in the numerical solution of the problem at hand.

Rotational compliance is not treated in this section, as it would unnecessarily complicate the exposition. Further, in both the conceptual and mathematical model, the contribution of the support rotation to tip deflection can simply be added to that due to beam deformation. Therefore, the solutions presented here are for a beam with its supported end perfectly fixed against translational and rotational motion. In practice, this sort of simplification must be carefully evaluated on a case-by-case basis.

We shall present two closely related methods for estimating discretization error. The first uses three grids; the second uses only two but requires an assumed value for the order of convergence. Both are based on *Richardson extrapolation* [5], which will now be explained. As in the prior section, it is assumed that the numerical value of the quantity of interest is related to the exact numerical solution by eq. (4) with higher order terms neglected (i.e., the numerical solution is in the asymptotic convergence regime). Assuming that w_h and h are known, eq. (4) then contains three unknowns, w_{exact}, A, and p. Richardson extrapolation is the process of computing w_{exact} by using eq. (4). The first version we shall apply simply entails writing it three times, using three pairs (w_h, h) based on three numerical solutions at different grid resolutions. This provides three independent equations that may be solved for the three unknowns. To be specific, writing eq. (4) three times, then eliminating A and w_{exact} leads to

$$\frac{w_2 - w_1}{w_3 - w_2} = \frac{h_1^p - h_2^p}{h_2^p - h_3^p} \tag{6}$$

where now the subscripts 1, 2, 3 refer respectively to the finest, intermediate, and coarsest numerical solutions for the SRQ of interest. This is a transcendental equation that may be solved for p by any suitable numerical method. Although certainly not necessary, in some cases it may happen that the grid refinement ratio is constant (i.e., $h_2/h_1 = h_3/h_2 = r$) in which case eq. (6) can be solved in closed form to yield

$$p = \ln\left(\frac{w_3 - w_2}{w_2 - w_1}\right)/\ln(r) \tag{7}$$

Once p is determined, eliminating A from the two finer mesh instances of eq. (4) leads to

$$w_{\text{exact}} = w_1 + \frac{w_1 - w_2}{(h_2/h_1)^p - 1} \tag{8}$$

This value of w_{exact} is a Richardson extrapolation based on three grids. [Although neither w_3 nor h_3 appears explicitly in eq. (8), they both implicitly affect p through either eq. (6) or eq. (7).] Equation (8) can be easily rearranged to provide an estimate of the numerical error in the fine-mesh solution w_1, by rewriting it as

$$w_{\text{exact}} - w_1 = \frac{\epsilon}{(h_2/h_1)^p - 1} w_1 \tag{9}$$

where
$\epsilon = (w_1 - w_2)/w_1$

For some purposes eq. (9) may be all that is needed to estimate the numerical error in the fine-grid solution. However, to standardize reporting of numerical error estimates, Roache [5] defined a grid convergence index (GCI) based on the right side of eq. (9), and it has been fairly widely adopted in the computational fluid dynamics community. Dividing eq. (9) by w_1 and multiplying by a factor of safety F_s (which Roache takes as 1.25 for "convergence studies with a minimum of 3 grids to...demonstrate the observed order of convergence..."), one obtains the GCI:

Table 2 Numerical Solutions for Tip Deflections

Grid Number	Number of Elements	h, m	w, mm
3	4	0.5	13.098739
2	8	0.25	13.008367
1	12	0.16666667	12.991657
Surrogate for exact solution	200	0.01	12.978342

$$\text{GCI} = F_s \frac{|\epsilon|}{(h_2/h_1)^p - 1} \tag{10}$$

Thus the GCI is a dimensionless indicator for the mesh convergence error relative to the finest-zoned solution. Specifically, when multiplied by the finest-zoned solution, it provides the width of an error band, centered on that solution, within which the exact solution is very likely to be contained. Its validity rests on the assumption that the numerical solutions from which p was determined are in the asymptotic convergence range. In terms of the GCI, the error band is $w_1(1 \pm \text{GCI})$. Equation (10) can be used for any discretization method with any order of spatial convergence. The only other requirement for eq. (10) is that $h_1 < h_2$. It is also recommended that successive grid refinement be greater than 1.3 (i.e., $h_2/h_1 > 1.3$).

The reasons for referring to the GCI-based interval as a *band* rather than a *bound*, and for applying the factor of safety, are the same: There is no guarantee that the converged numerical solution will fall within the band, just a high likelihood. The converged solution could fall outside the band for various reasons mostly related to the numerical solutions not being in the asymptotic convergence regime [which means that the higher order terms in eq. (4) were not negligible]. A strong piece of evidence that the numerical solution is in the asymptotic regime is that the inferred order of convergence is close to the theoretical one.

The foregoing analysis will now be applied to numerical solutions of the problem at hand, which are summarized in Table 2.

Using the first three of these displacements in eq. (6) and solving for p yields $p = 2.00256154$.

Since this is so close to the theoretical value of 2, the solutions are judged to be in the asymptotic convergence regime. From eq. (10) with $F_s = 1.25$, we have GCI = 0.00128381. The error band defined by this GCI about the fine-zoned solution w_1 is (12.9750, 13.0083) mm. The last line in Table 2 lists a 200-zone solution, which we regard as a surrogate for the exact numerical solution, and note that it does indeed fall within the GCI-defined interval.

The recommended method of error estimation is based on three numerical solutions as just described. If resources are insufficient to permit three numerical

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solutions, it is still possible to estimate discretization error based on only two solutions, although this approach is to be avoided if possible. When only two solutions are available, then rather than computing the *observed* order of accuracy p, the *theoretical* value must be used. This is what Richardson did when he first proposed his extrapolation, and accordingly, the two-grid process is referred to as *standard* Richardson extrapolation. Having thus selected a value for p, there are only two remaining unknowns in eq. (4), and the two available instances of it combine once again to yield eq. (8). The GCI can still be defined by eq. (10), similarly to the three-grid case, but now Roache recommends a larger factor of safety, viz., $F_s = 3$, to account for the greater uncertainty in the order of accuracy. Using the two finer grid solutions for displacement in eq. (10) with $p = 2$ and $F_s = 3$ yields the two-grid GCI = 0.003087, larger than the three-grid value in this case by almost exactly the ratio of the safety factors 3/1.25. Naturally the surrogate exact solution still falls in the wider error band defined by the two-grid GCI. If instead we ignore solution 1, regard grid 2 as the finer solution, and use grids 2 and 3, the two-grid GCI relative to w_2 is 0.00694727, more than twice the one based on the finer two solutions, and the inferred error band is correspondingly wider. As a final observation on GCI, note that according to eq. (10), with mesh doubling, second order spatial convergence, and a factor of safety of 3, the two-grid GCI reduces simply to the fractional error $|\epsilon|$.

An immediate application of these calculation verification results is to assess the adequacy of the grid resolution used when comparing predictions of structural response to experimental measurements. As mentioned in para. 5.2, in this example we specify that estimated numerical error be no more than 2% of the 10% validation requirement, or 0.2%. Thus the three-grid GCI of 0.001284 or 0.1284% implies a numerical error in tip deflection no greater than 0.13% when 12 elements are used. While this would be adequate, for added safety 20 elements with length 0.100 m will be used for validation predictions.

8 VALIDATION APPROACH 1

Validation Approach 1 considers the case where uncertainty data are not available, and suggests an approach whereby subject matter experts are relied upon to provide estimates of the expected uncertainty. The area metric is used to assess the validity of the model. The value of an unknown model parameter is first estimated from a single, special test, followed by the model prediction, measurement of the beam response in a validation experiment, and finally the validation assessment.

8.1 Obtaining the Model Result

The numerical simulation requires a value for the rotational flexibility, f_r, at the clamped end of the beam. One could assume that f_r is negligibly small (i.e., the rotational stiffness is infinite), but engineering experience has shown this to be a poor assumption in most applications. The parameter f_r cannot be measured directly. However, it can be determined using a procedure that combines experimental measurements on a specially constructed beam and computational modeling of that beam. This procedure will be discussed in more detail in section 9. For Validation Approach 1, this procedure, when applied to a single, special test article, yielded a value of $f_r = 8.4 \times 10^{-7}$ rad/N·m.

All the information necessary to create a finite element model of the beam is now available. Specifically, this includes

(a) the geometric and material characteristics of the beam as listed in para. 6.3.

(b) the specified load (over the outer half of the length).

(c) the parameters needed to characterize the boundary conditions.

(d) the number of finite elements needed to demonstrate that the numerical solution error is negligible, as determined by the calculation verification procedure in para. 7.2. The tip deflection predicted by the model is

$$w^{\text{mod}} = -14.2 \text{ mm} \tag{11}$$

8.2 Experimental Data

To obtain validation data, we construct a single beam with the dimensions and constraints listed in para. 6.3. After constructing the beam, it is embedded into the wall. The beam is instrumented with a displacement transducer at the tip to measure deflection. The measurement system is initialized to record displacement relative to the initial, gravitationally deformed configuration. The beam is then loaded by placing a 500 N·m load on the right half of the beam. The resulting tip deflection is recorded as

$$w^{\text{exp}} = -15.0 \text{ mm} \tag{12}$$

8.3 Validation Assessment

The primary activities in validation assessment are model accuracy assessment by comparison with experimental validation data, and determination whether the validation accuracy requirement is satisfied. In addition, it is commonly necessary to assess the accuracy of a model's predictions for conditions where experimental data are not available (i.e., where extrapolation of the model is required). The latter activity, however, is regarded as beyond the scope of this Standard.

As stated in para. 5.3, Validation Approach 1 specifies that the model-predicted tip deflection of the beam be

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within 10% of the respective experimental measurements, as measured by the area metric. We describe the validation comparison procedure in detail in the following paragraphs. Before doing so, however, we emphasize again that the details of the validation requirements should be decided before performing the validation experiments and the model predictions. One of the reasons for this emphasis is that it focuses on expectations of the model for the particular application of interest, as opposed to what accuracy is determined in the validation assessment.

In Validation Approach 1, uncertainties are estimated by subject matter experts (SMEs), who are each asked to provide their estimate of the range within which all practical experimental or model outcomes are expected to fall. Specifically (see Fig. 4), each SME is asked to provide the half-width, Δ, of the full uncertainty interval, which is then interpreted to be equal to three standard deviations in an assumed normal PDF. For this demonstration, the estimates provided by the respective SMEs based on their experience with related problems are

$$\Delta^{\text{exp}} = 0.75 \text{ mm}, \Delta^{\text{mod}} = 0.71 \text{ mm} \tag{13}$$

From this, assumed normal PDFs and CDFs for the experiment and model can be constructed with the following parameters

$$\begin{aligned} \mu^{\text{exp}}(=w^{\text{exp}}) &= -15.0 \text{ mm}; \sigma^{\text{exp}} = \frac{0.75}{3} = 0.25 \text{ mm} \\ \mu^{\text{mod}}(=w^{\text{mod}}) &= -14.2 \text{ mm}; \sigma^{\text{mod}} = \frac{0.71}{3} = 0.24 \text{ mm} \end{aligned} \tag{14}$$

These inputs are now used to compute the validation metrics. As shown in Fig. 8, the area between the CDFs was computed to be 0.8 mm, so according to eq. (1) the area metric is $M_A^{SRQ} = 5.3\%$. (In keeping with the properties listed in para. 5.3 and that the two CDFs cross only near one extreme where they are practically equal, this value is almost exactly the absolute relative difference of the means.) Because the validation metric falls within the 10% requirement, the model is assessed as valid.

9 VALIDATION APPROACH 2

Validation Approach 2 considers the case where uncertainty data are available, and employs a straightforward probabilistic analysis to relate model input uncertainties to the model output uncertainty. The area metric is then used to assess the validity of the model.

9.1 Validation Experiments

For validating a model, multiple replications of the validation experiment are highly recommended. This will account for inevitable variations that exist in the test article fabrication, experimental setup, and measurement system. For any application, the questions that must be addressed and answered in the V&V Plan include the following:

(a) How many experiments should be conducted?

(b) Should different technicians or subcontractors assemble the system to be tested?

(c) Should different experimental facilities conduct the experiment?

These types of questions must be answered on a case-by-case basis.

In this example the data used to characterize the response of the system come from validation experiments on 10 nominally identical beams, constructed with the nominal dimensions listed in para. 6.3.

After constructing each beam, the same procedure described in para. 8.2 concerning measurements relative to the gravitational equilibrium conditions of the beam is used. The uncertainty in the response of the beam is due to random and systematic uncertainty in the multiple experimental measurements as well as variability in the properties of the test article. In this example, properties variations are assumed to be confined to the modulus of elasticity in the material used to construct each beam, and the support flexibility. These independent sources of uncertainty can be separated using statistical design of experiment techniques [6, 7].

Uncertainty in the experimental measurements will be due to a number of random and systematic uncertainties. Some examples of random uncertainty are transducer noise, attachment of individual transducers, and setup and calibration of all of the instrumentation for each test. Examples of systematic uncertainties are calibration of the transducers, unknown bias errors in the experimental procedures, and unknown bias errors in the experimental equipment. The measured displacements in the validation experiment are denoted as $w_i^{\text{exp}}, i = 1, \ldots, 10$. The measurements are given in Table 3.

These data can be used to compute the sample mean and standard deviation of the experimental tip deflections:

$$\begin{aligned} \bar{w}^{\text{exp}} &= \frac{1}{10}\sum_{i=1}^{10} w_i^{\text{exp}} = -15.4 \text{ mm} \\ \sigma^{\text{exp}} &= \sqrt{\frac{1}{10-1}\sum_{i=1}^{10}\left(w_i^{\text{exp}} - \bar{w}^{\text{exp}}\right)^2} = 0.57 \text{ mm} \end{aligned} \tag{15}$$

For use in the area metric, an empirical CDF can be constructed from the validation experiment data. The data listed in Table 3 are sorted in ascending order and a probability value of i/N is assigned to each of the data points. The empirical CDF is shown in Fig. 9. This CDF is "stair-stepped" because of the finite number of data points, each with a single associated probability.

9.2 Model Uncertainty Quantification

Uncertainty quantification provides the basis for quantifying and understanding the effect of uncertainties in experimental data and computational predictions,

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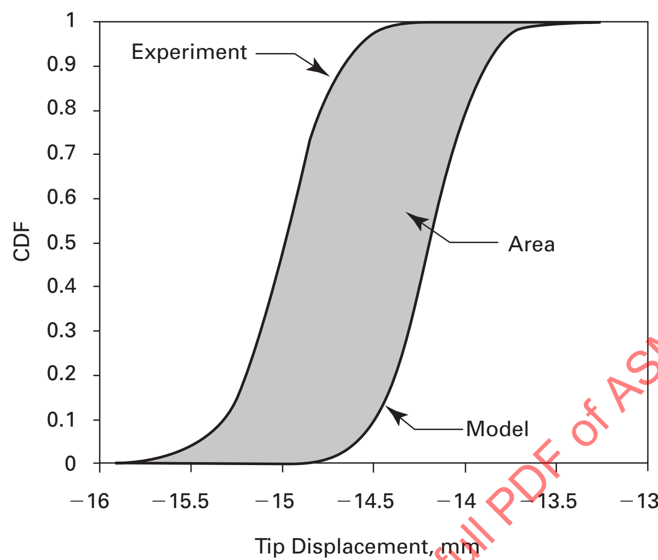
Fig. 8 Area Between the Experimental and Computed CDF



GENERAL NOTE: Used in area metric.

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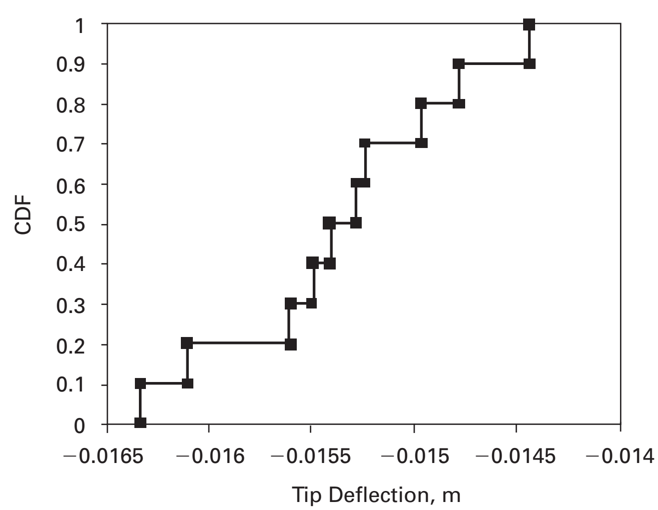
Table 3 Measured Beam-Tip Deflections From the Validation Experiments

Test Number, i	Tip Deflection, w_i^{exp}, mm
1	−16.3
2	−15.5
3	−16.1
4	−14.8
5	−14.4
6	−15.0
7	−15.3
8	−15.4
9	−15.6
10	−15.2

Fig. 9 Empirical CDF of the Validation Experiment Data



for judging the adequacy of the model for its intended use, and for quantifying the predictive accuracy of the model. As compared to traditional deterministic analysis, quantifying the effects of uncertainties requires additional effort to collect and characterize input data, to perform the uncertainty analysis, and to interpret the results.

Uncertainties enter into a computational simulation from a variety of sources, such as variability in input parameters due to inherent randomness, lack of or insufficient information pertaining to models or model inputs, and modeling assumptions and approaches. For example, a material property such as modulus of elasticity will vary from sample to sample because of the inherent variability in manufacturing of the material. Loadings will often be random due to the uncertainty of environmental factors such as wind, wave height, and as-built conditions. Uncertainties like these are beyond our ability to control and are referred to as inherent or irreducible. Recognizing this, the approach is to assess their effects on the results of the simulation and the performance of the system.

To facilitate the numerical representation and simulation of uncertainties, some type of uncertainty model and uncertainty quantification method is required. A well-known approach to modeling uncertainties is based on the theory of probability using what are called random variables. In simple terms, a random variable is a function that relates the possible values of the variable to the corresponding probability of occurrence. In this application, uncertainty is manifested in the material modulus and constraint flexibility, leading to uncertainty in the SRQ of interest, namely the tip deflection.

9.3 Random Variability in the Material Modulus

One source of variability in the validation experiments is the modulus of elasticity, E, of the material used in the beam. This variability is due to inherent variations in the material, and these variations cannot be eliminated in the production of the beam or in the validation experiment. Therefore, we measure the material variability from replicate experiments on coupon samples taken from the same material as used in the beam experiments. This variability will then be included in the beam model, as discussed later. A set of 10 sample measurements is given in Table 4.

Table 4 Test Measurements of the Modulus of Elasticity, *E*

Test	Modulus, *E*, GPa
1	69.1
2	68.8
3	74.4
4	72.6
5	72.9
6	67.5
7	74.1
8	68.3
9	71.0
10	63.2

Using the 10 measurements of E given in Table 4, the sample mean and standard deviation are computed to be 70.2 GPa and 3.5 GPa, respectively. A histogram illustrating the variability in E is shown in Fig. 10, illustration (a). For use in the computational model, it is convenient and usually acceptable to represent the histogram with a continuous function. Because it fits the data well, a Gaussian distribution with the same mean and standard deviation is selected, and this is shown in both PDF and CDF form in Fig. 10, illustration (b). A different distribution (e.g., lognormal or Weibull) also could be selected if it fit the data better.

9.4 Random Variability in Support Flexibility

To characterize the random variability in the support flexibility, f_r, 20 experiments were performed using a specially made and instrumented beam. This special beam is not tapered and not subjected to the same loading specified for the beams used in the validation experiments. The attachment of this special beam, however, closely replicated the method of attachment of the beams used in the validation experiments such that the measured variability would be similar. The beam was designed so that very high confidence was attained in all of the assumptions made in the underlying mathematical model. For example, all dimensions were accurately measured and used in the special computational model mentioned below, and the elastic modulus was measured in a special coupon test using the same material as the beam. During each experiment a carefully calibrated transverse load was applied, and the rotation at the supported end of the beam was inferred as follows: a high fidelity computational model was run with sufficient spatial resolution that a converged numerical solution was guaranteed, so that high confidence was attained in the simulation results. The only parameter in the model left unspecified was f_r. For each of the experiments conducted, a measurement was made for the tip deflection at the free end of the beam. Then, the f_r parameter was adjusted so that the computational result matched the experimental measurement for the tip deflection at the free end of the beam. (This is a trivial example of an optimization procedure usually referred to as *parameter estimation*.) The results from these experiments are listed in Table 5.

Using the 20 estimates of f_r given in Table 5, the sample mean and standard deviation are computed to be 8.4×10^{-7} rad/N·m and 4.3×10^{-8} rad/N·m, respectively. The variability of f_r shown in the table is a combination of experimental measurement uncertainty and variability

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Fig. 10 Random Variability in Modulus, *E*, Used in the Computational Model

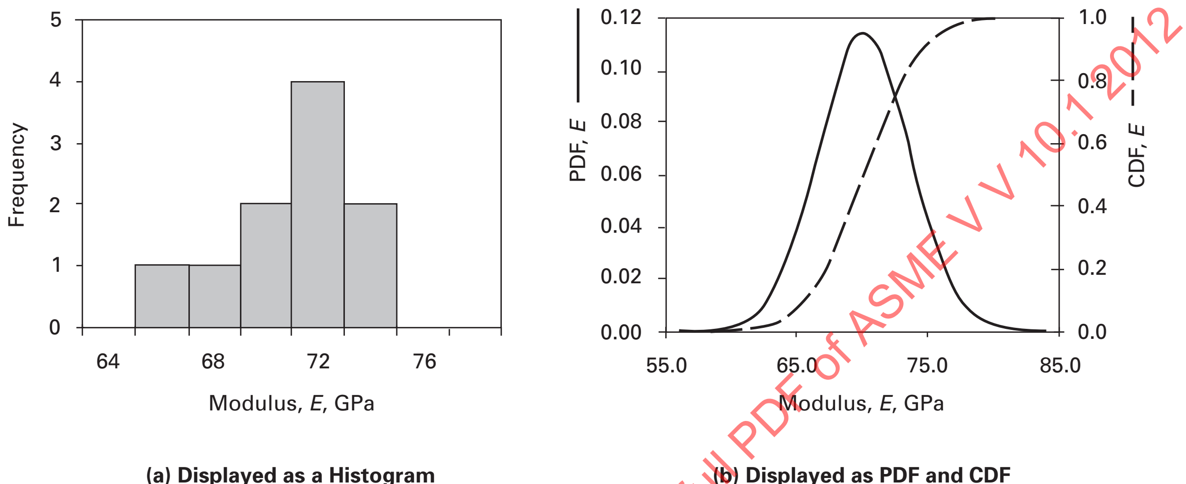

Table 5 Test Estimates of the Support Flexibility

Test	f_r, rad/N·m × 10^{-7}
1	8.5
2	8.2
3	8.1
4	8.4
5	7.8
6	7.7
7	8.8
8	8.6
9	8.6
10	8.1
11	8.6
12	8.8
13	8.5
14	8.0
15	9.3
16	8.1
17	9.3
18	8.1
19	8.3
20	8.3

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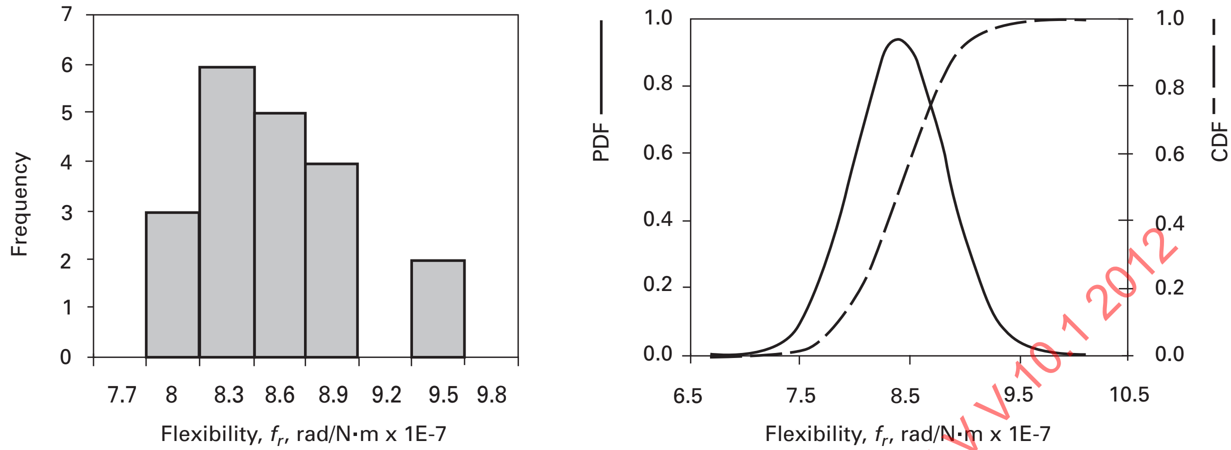
Fig. 11 Random Variability in Support Flexibility, f_r, Used in the Computational Model



(a) Displayed as a Histogram **(b) Displayed as PDF and CDF**

in repeatedly attaching the special beam to the test fixture. A histogram illustrating the variability in f_r is shown in Fig. 11, illustration (a). Again, a Gaussian distribution is selected. A different distribution (e.g., lognormal or Weibull) also could be selected if it fit the data better. Both the PDF and the corresponding CDF are shown in Fig. 11, illustration (b).

9.5 Uncertainty Propagation

The process of propagating input uncertainties through the computational model is called uncertainty propagation. This process is illustrated in Fig. 12, where the input uncertainties are propagated through the beam model. If all inputs had been treated as deterministic (single valued), the beam tip displacement would also have been single valued. It is the uncertainty in all of the input parameters, and their impact on computational model output, that yields the uncertainty in the beam tip displacement shown in the figure.

There are many different techniques for propagating input uncertainties through a computational model. These different methods have been developed primarily to propagate uncertainties in an efficient and accurate manner for different situations (e.g., static vs. dynamic responses) and for different purposes (e.g., computing overall statistics such as the mean and standard deviation vs. computing extremely small probabilities).

A well-known technique for performing uncertainty propagation is Monte Carlo simulation, which is a straightforward random sampling method. In Monte Carlo simulation, a random sample is taken from each of the input distributions. (The distributions may be Gaussian or not.) This sample is then used in the computational model to produce one output sample of the response. This process is repeated a number of times, the number of which typically depends on the magnitude of the response probability of interest. For example, if an outcome with a low probability of occurrence is of interest, then a large number of Monte Carlo samples are required to estimate this probability accurately. Once the Monte Carlo simulation is finished, the output samples are processed to produce the CDF of the tip deflection.

The CDF of the beam tip deflection resulting from a Monte Carlo simulation is shown in Fig. 13. The CDF appears as a relatively smooth curve because a large number of Monte Carlo samples were used. The computed mean and standard deviation for the tip displacement are −14.1 mm and 0.65 mm, respectively.

9.6 Validation Assessment

Figure 14 shows the CDFs of the computed and measured tip-deflection of the beam. The area between them, indicated by the shaded regions in the figure, is calculated to be 1.3 mm. Validation Approach 2 specifies that the area normalized by the absolute experimental mean be less than 10%. This is calculated to be 1.3/15.4 = 8.4%. Thus, the validation requirement for tip displacement is satisfied.

If the validation requirement had not been met, possible next steps could include the following:

(a) obtaining additional validation test data

(b) obtaining additional data on the model input uncertainties

(c) modifying the model and/or experiment to correct any suspected deficiencies

(d) relaxing the validation requirements

Although obtaining additional data is always desirable, (a) and (b) may or may not decrease the metric, depending on how they change the relative shapes of the CDFs. The item listed in (c) always should be performed when the validation requirements are not satisfied.

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Fig. 12 Input Uncertainty Propagation Process

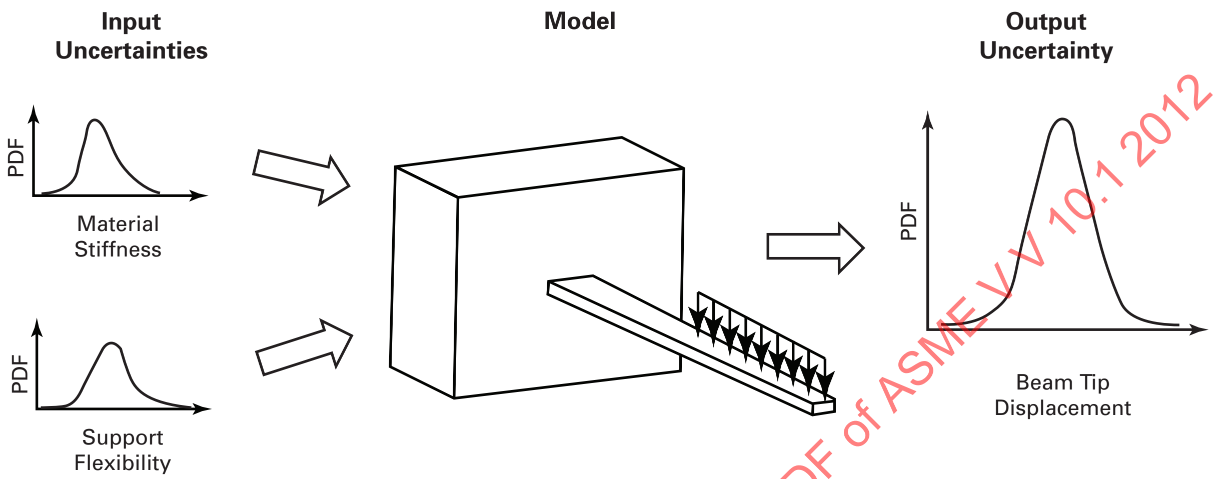
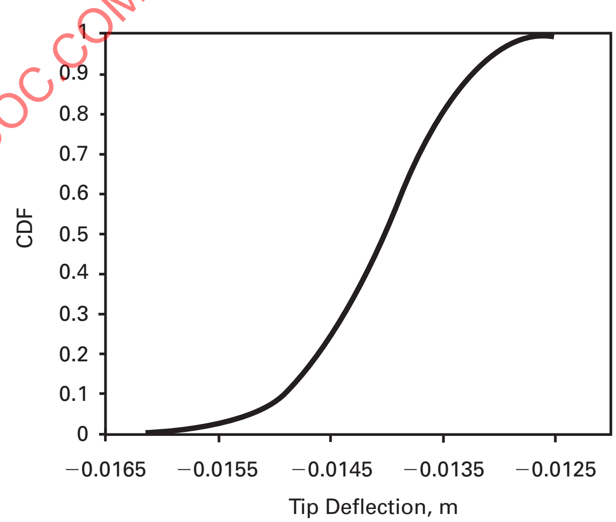


Fig. 13 Computed CDF of Beam Tip Deflection



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