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TECHNICAL REPORT

colour inside

Electromagnetic compatibility (EMC) –
Part 4-40: Testing and measurement techniques – Digital methods for the measurement of power quantities of modulated or distorted signals

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IEC TR 61000-4-40:2020-03(en)

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ELECTROMAGNETIC COMPATIBILITY (EMC) –

**Part 4-40: Testing and measurement techniques –
Digital methods for the measurement of power quantities
of modulated or distorted signals**

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IEC TR 61000-4-40, which is a Technical Report, has been prepared by subcommittee SC77A: EMC – Low frequency phenomena, of IEC technical committee TC 77: Electromagnetic compatibility.

The text of this Technical Report is based on the following documents:

Enquiry draft	Report on voting
77A/1055/DTR	77A/1065/RVC

Full information on the voting for the approval of this technical report can be found in the report on voting indicated in the above table.

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This document has been drafted in accordance with the ISO/IEC Directives, Part 2.

A list of all parts in the IEC 61000 series, published under the general title *Electromagnetic compatibility (EMC)*, can be found on the IEC website.

The committee has decided that the contents of this document will remain unchanged until the stability date indicated on the IEC website under "http://webstore.iec.ch" in the data related to the specific document. At this date, the document will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

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INTRODUCTION

IEC 61000 is published in separate parts, according to the following structure:

Part 1: General

General considerations (introduction, fundamental principles)

Definitions, terminology

Part 2: Environment

Description levels

Classification of the environment

Compatibility levels

Part 3: Limits

Emission limits

Immunity limits (in so far as they do not fall under the responsibility of the product committees)

Part 4: Testing and measurement techniques

Measurement techniques

Testing techniques

Part 5: Installation and mitigation guidelines

Installation guidelines

Mitigation methods and devices

Part 6: Generic standards

Part 9: Miscellaneous

Each part is further subdivided into several parts, published either as International Standards, Technical Specifications or Technical Reports, some of which have already been published as sections. Others are and will be published with the part number followed by a dash and a second number identifying the subdivision (example: IEC 61000-6-1).

This document gives the rationale for the assessment of electrical power quantities (RMS voltage, RMS current and active power) under non-stationary conditions. It explains and compares two digital methods that can be used in digital measurement instrumentation to either average or filter the signals when measuring fluctuating loads, and algorithms for the realization of both methods. The examples relate to 50 Hz or 60 Hz power systems because power quantity assessments are predominantly required for these systems.

The digital averaging or integration algorithm is evaluated for fluctuating, or non-stationary, conditions, as is a digital filtering algorithm that emulates the traditional analogue power meter.

This document aims to illustrate the application of the two measurement algorithms given above to characterize existing, and commonly found, non-stationary loads, which have been selected to help interpret the measurement results obtained using both algorithms.

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ELECTROMAGNETIC COMPATIBILITY (EMC) –

Part 4-40: Testing and measurement techniques – Digital methods for the measurement of power quantities of modulated or distorted signals

1 Scope

This part of IEC 61000, which is a Technical Report, deals with the assessment of electrical power quantities (RMS voltage, RMS current and active power). It explains and compares two digital algorithms suitable for power quantity measurements in fluctuating or non-periodic loads. The examples are from 50 Hz or 60 Hz power systems.

This document does not attempt to cover all possible digital implementations of the algorithms used for power quantity assessment in fluctuating loads, for example in the context of the EMC assessment described in several IEC documents. Rather, it compares averaging with one of the filtering algorithms. This document aims to highlight some examples of applications that illustrate how the presented algorithms work. Further, guidance is given for quantifying the accuracy of each approach.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the cited edition applies. For undated references, the latest edition of the referenced document applies, including any amendments.

IEC TR 61000-1-7:2016, *Electromagnetic compatibility (EMC) – Part 1-7: General – Power factor in single phase systems under non-sinusoidal conditions*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in IEC TR 61000-1-7 apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- IEC Electropedia: available at http://www.electropedia.org/
- ISO Online browsing platform: available at http://www.iso.org/obp

4 General

IEC TR 61000-1-7:2016, 3.1, defines the root-mean square (RMS) value of a time-dependent quantity as a positive square root of the mean value of the square of the quantity taken over a given time interval.

IEC TR 61000-1-7:2016, 5.1.4, further states that the RMS value of the voltage U (current I) is defined as the positive square root of the mean value of the square of the voltage $u(t)$ (current $i(t)$) taken over an integer number of periods kT of the AC power supply system:

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$$U = \sqrt{\frac{1}{kT} \int_{\tau}^{\tau+kT} [u(t)]^2 \, dt} \quad (1)$$

$$I = \sqrt{\frac{1}{kT} \int_{\tau}^{\tau+kT} [i(t)]^2 \, dt} \quad (2)$$

where

T is the reciprocal of the reference fundamental frequency;

k is an integer number;

τ is the time when the measurement starts.

Similarly, the active power is defined in IEC TR 61000-1-7 as the mean value, taken over an integer number of periods kT, of the instantaneous power $p(t) = u(t)\ i(t)$:

$$P = \frac{1}{kT} \int_{\tau}^{\tau+kT} p(t) \, dt \quad (3)$$

In digital instrumentation, the assessment of the RMS value of voltage or current is performed by first obtaining the squares of the sampled values of the signal. Similarly, for the assessment of active power the products of each pair of the instantaneous voltage $u(t)$ and current $i(t)$ samples are obtained. Then the instrument performs the integration of the squared or multiplied samples over the measurement time interval. To adhere to IEC TR 61000-1-7, the measurement time interval is normally set to an integer multiple of the period of the power system fundamental frequency, but many instruments permit the user to select arbitrary time intervals. Further, for AC power systems, such as 50 Hz or 60 Hz public supply networks, the values of non-active power and apparent power can be derived from the obtained RMS values of current, voltage and active power.

For a sinusoidal signal, the multiplication of voltage and current, or the squaring operation, gives a function whose period is half of the period of the sine wave. This function contains a zero-frequency (DC) component that is equal to the active power or the square of the RMS value. In addition to the desired DC component, there is also an AC component at twice the frequency of the sine wave that it is essential to remove, or at least heavily attenuate, to retrieve the DC value.

Historically, instruments for the measurement of power quantities were implemented in an analogue form, using certain characteristics of thermal, magnetic or electrical components. In moving iron meters, for example, the squaring step is realised through a magnetic force applied to a vane made of iron. This magnetic force, proportional to the square of the current, is generated by a current flowing in a coil. When measuring a sinusoidal signal, the force oscillates at twice the frequency of the sine wave and causes the vane with its attached pointer to vibrate at the same frequency. To produce a stable reading, the assembly is mechanically damped (a smoothing function). The damper is analogous to a low-pass filter, decreasing oscillations caused by the alternating current. The measured RMS value is indicated on a non-linear scale devised according to the electromechanical properties of the meter.

Since the RMS value of an electric signal represents a heating effect, another analogue approach, implemented using thermal converters, is to heat a resistor (heater) with a voltage or a current applied across its terminals. The heater temperature is then measured with a thermocouple producing a DC voltage proportional to the square of the RMS current passing through the resistor. The thermal medium of the thermal converter smoothes the temperature measured by the thermocouple. Thus this thermal smoothing effect also behaves like a low-pass filter.

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Further modifications of these techniques, with two coils (electrodynamic wattmeters) or two or more thermal converters (thermal wattmeters and thermal power comparators), enable the measurement of active power in stationary conditions.

In analogue devices the processed signal is smoothed with the internal time constant of the meter, which allows for a steady reading of the result to be made for stationary input signals. Even with fluctuating loads, when the needle of the meter is not completely stable, it is often possible to determine the average value by observing where the variation is centred.

To obtain a similar result, the manufacturers of digital instruments usually add digital filtering to their measuring algorithms, which helps stabilize and/or average the readings. In the simplest form, the filtering is based on averaging over multiple periods of the signal. As the power system frequency is usually quite accurate, digital measurement instruments often use a constant measurement time interval corresponding to a multiple of the nominal period of the power system. For example, the 200 ms time interval specified in IEC 61000-4-7 corresponds to 10 cycles of a 50 Hz signal and 12 cycles of a 60 Hz signal. Further filtering can be obtained by using a digital implementation of the low-pass filter function. For example, in IEC 61000-4-7 a low-pass filter with a 1,5 s time constant was selected, partly because it reproduces the typical behaviour of a moving coil instrument.

When the period of the signal does not correspond to the nominal 50 Hz or 60 Hz power system frequency, readings from instruments that use a constant measurement time interval often show fluctuating results. For example, multi-cycle symmetrical control (MCSC) used in water heaters produces current waveforms with periods that are longer than the fundamental frequency period of the power system voltage feeding the device. Additionally, these MCSC controls can vary the control cycle from one instant to another as required, to maintain water temperature under different flow conditions. Another example is fluctuating loads, such as refrigerators, where compressor motors can be energised at random times, producing non-periodic currents. It is also noted that supply voltage frequency variations are common, for example, in isolated power systems having no electrical connections to a large interconnected system, such as is common in remote communities served by small generation sources.

To characterise the performance of various devices, many documents require the determination of reference current or power. Additionally, for various voltage quality assessments, specific measurement time intervals have been defined by IEC documents, such as half-cycle, 10 or 12 cycles for 50 Hz or 60 Hz power systems, 3 s, 10 min and 2 h.

Stable readings are often a prerequisite in order to obtain comparable results. In the case of fluctuating loads these are sometimes difficult to achieve using conventional voltage, current and power meters. In these situations the current and voltage can be recorded by data loggers and post-processed using, for example, spreadsheet software. Smoothing functions corresponding to the fluctuation rates can then be implemented as required. As data logger recordings are often limited in their duration, fast-settling filters are desirable.

This document compares one averaging and one filtering algorithm used to assess the power quantities for four typical groups of waveforms. For simplicity, the amplitude of the current waveform used in the study was adjusted to give an RMS current of 1 A. The voltage was also adjusted at the appropriate level to obtain an active power of 100 W.

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5 Modulated sine waveforms used in this document to compare measurement algorithms

5.1 General

For an ideal sine wave with constant amplitude and frequency, most measurement algorithms would produce accurate results within the capabilities of the measuring instrument. The situation becomes more complicated if the sine wave is randomly modulated and/or contains distortion, as found in uncontrolled environments. Some examples of current waveforms produced by real-life equipment that challenge the assessment of power quantities are described in Clause 5. There exist even more complex situations that are not addressed in this document.

5.2 Half-wave rectification

Half-wave rectification occurs when the equipment is connected only during one polarity of the cycle (e.g. the current of a hair dryer, illustrated in Figure 1). In this case there are, in principle, two possible measurement time intervals of interest.

The half-wave rectified current waveform is asymmetrical with a period of around 16,667 ms in a 60 Hz power system. Therefore, the first appropriate measurement time interval to select is one or more whole cycles of the power line frequency. If the current varies, a stable measurement can only be obtained, if desired for the application, by the use of a measurement time interval containing a larger number of periods.

Secondly, to assess instantaneous voltage fluctuation $d(t)$, as required by IEC 61000-4-15 for flicker assessment, the measurement time interval should be equal to one half-cycle. Whilst measurement of the power of this waveform in half-cycle intervals is not usually used for general power quantity assessment, the analysis of this waveform over a half-cycle interval can highlight the need for correct synchronisation for the averaging measurement algorithm.

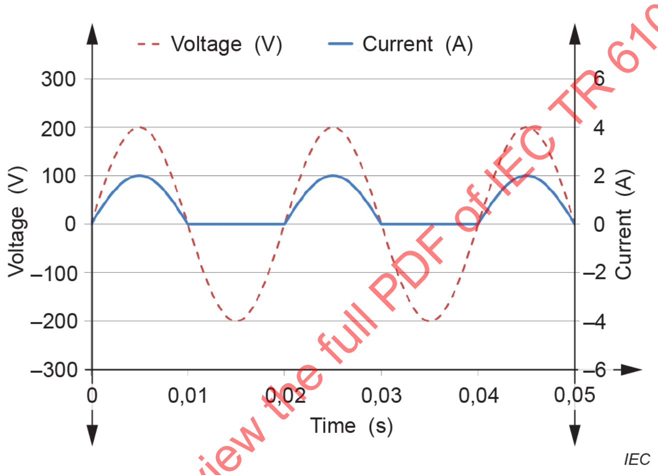

Figure 1 – Typical resistive load current and supply voltage waveform of half-wave rectification

5.3 Full-wave rectification

Full-wave rectification is used in DC power supplies of various common electronic devices. An interesting feature of these power supplies is the concentration of current conduction near the peak of the voltage. When the energy is concentrated in a small part of the period, a larger number of samples covering that part is required to reduce the instrumental errors.

For the purposes of this document, voltage and current in a real item of equipment based on full-wave rectification were measured using a 100 kHz sampling frequency. The results were then normalised to give an RMS current of 1 A and a power of 100 W (see Figure 2).

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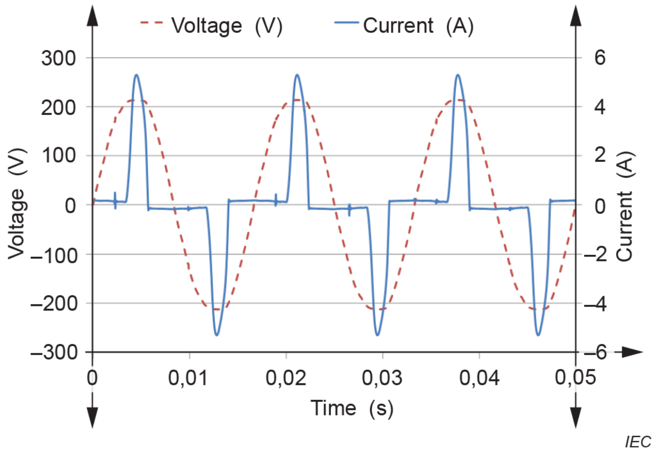
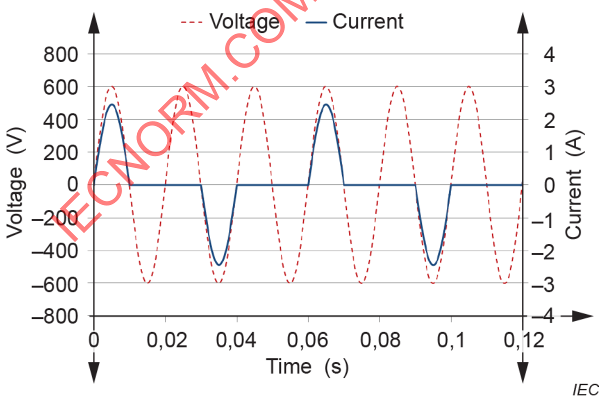


Figure 2 – Typical full-bridge rectifier current and supply voltage waveforms

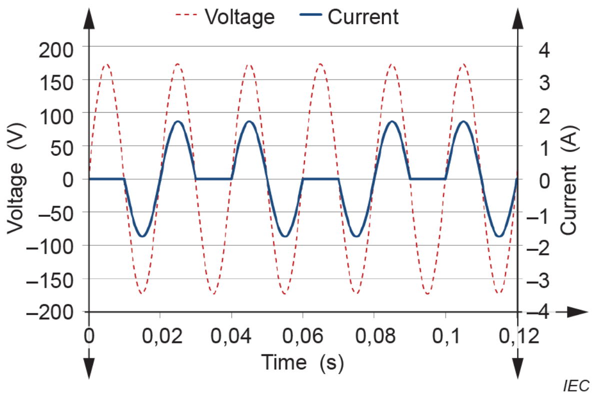
5.4 Multi-cycle symmetrical control

The multi-cycle symmetrical control (MCSC) technique enables regulation of the power delivered to a load. An MCSC circuit connects or disconnects the load for one or more half-cycles of the power system frequency. Two basic patterns, which can be combined to give other waveform patterns, are illustrated in Figure 3. In the first pattern the load is connected during one half-cycle per each three consecutive half-cycles of the mains frequency. This pattern is called 1/3 MCSC. Conversely, in the second pattern the load is connected during two half-cycles per each three consecutive half-cycles. This is the 2/3 MCSC pattern. Other commonly used patterns are 1/2 MCSC and 1/5 MCSC. Here "symmetrical" means that there is an equal number of half-cycles in the positive and negative parts of the repeating waveform pattern; this is necessary to avoid direct currents (DC), which are undesirable for distribution systems.

Each of the two patterns illustrated in Figure 3 is periodic, with a period equal to three cycles of the mains frequency. Equipment controlled by MCSC can produce varying currents having long periods, up to several seconds, with intermediate constant power levels for fractions of the overall control cycle. As patterns are combined to maintain the desired temperature, control cycles can vary from one second to the next. Only the two basic patterns are studied in this document with the aim of showing how these signals can be assessed with various measurement algorithms.



1/3 MCSC



2/3 MCSC

Figure 3 – Current and voltage patterns in an MCSC circuit, (left) 1/3 MCSC and (right) 2/3 MCSC

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5.5 Random on-off control

Some control circuits, such as those used in thermostats, turn equipment on and off during several seconds as required by their application. For example, a high-pressure die-casting machine in an industrial environment would typically turn its motor on for approximately 20 s during the moulding process and then disconnect it until the start of the next process. Other examples include refrigerators and older air conditioning systems with motors energised at random times.

When the application requires the measurement of an average power, or current, of such devices over a long period of time, the 10 min averaging technique suggested in IEC 61000-4-30 to obtain the power quantities at the point of common coupling (PCC) can be sufficient. To illustrate both algorithms described in Clause 6, a 50 Hz current with periods randomly varying between 1 min and 2 min was artificially generated. Figure 4 shows the RMS values of a signal alternating between 0 A and √2 A, the average of which gives a result of 1 A when evaluated over 30 min intervals, assessed using 10-cycle contiguous RMS windows.

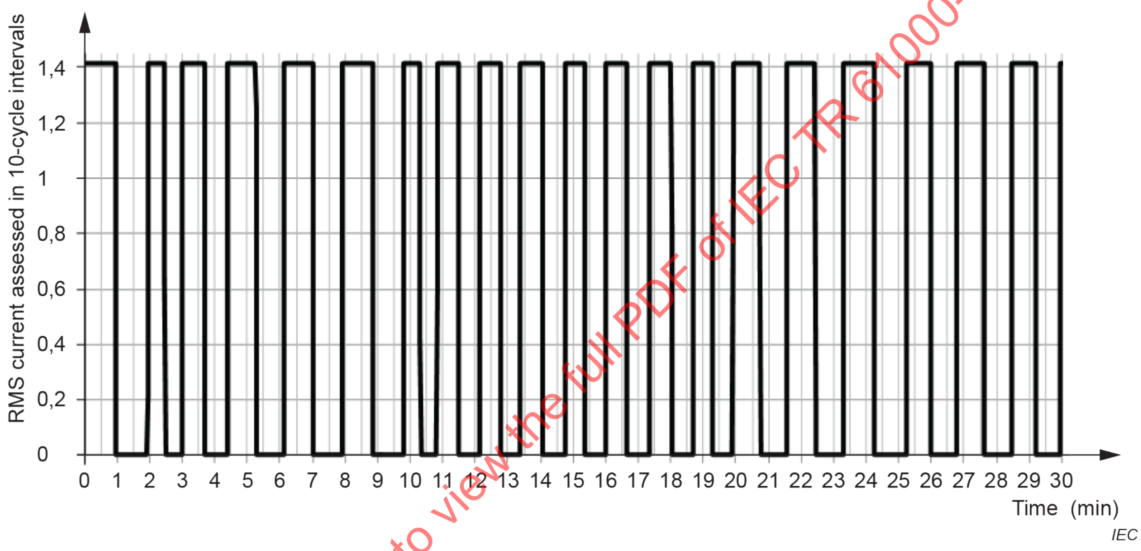

Figure 4 – Amplitude of 50 Hz current with on and off periods varying within a 1 min to 2 min range

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6 Measurement algorithms

6.1 General

The focus is set on two very commonly used groups of smoothing algorithms, averaging and filtering algorithms. Numerous filtering algorithms exist, but it is not the aim of this document to cover all of them. Instead, the aim is to explain the limitations of the two major algorithm categories.

6.2 Averaging algorithms

6.2.1 General

Digital measurement systems approximate the integral in the definition for active power in IEC TR 61000-1-7, by summing the instantaneous power samples obtained during the selected measurement interval. The sum is then divided by the number of samples N contained in the measurement time interval $N\Delta t$. The result is a time-dependent function which is the average of the instantaneous power samples over the selected measurement time interval. A similar algorithm is used for calculation of the RMS values of voltage and current.

$$P(t)=\frac{1}{N}\sum_{n=0}^{N-1}u(t-n\ \Delta t)\cdot i(t-n\ \Delta t) \tag{4}$$

$$U(t)=\sqrt{\frac{1}{N}\sum_{n=0}^{N-1}u^2(t-n\ \Delta t)} \tag{5}$$

$$I(t)=\sqrt{\frac{1}{N}\sum_{n=0}^{N-1}i^2(t-n\ \Delta t)} \tag{6}$$

For stationary signals, and when the measurement interval equals an integer number of periods of the AC power supply:

$$N\Delta t = kT\ , \tag{7}$$

the averaging algorithm returns a constant reading. In many other cases, when the energy contained within each measurement interval is not constant, the averaging algorithm reflects this by returning fluctuating readings.

6.2.2 Performance of the averaging algorithm

As follows from Formulae (4) to (6), subject to instrumental errors which can be caused, for example, by insufficient number of samples N, the averaging algorithm always returns a true value of the average energy contained in the measurement period. Therefore, when the signal to be measured varies throughout the measurement interval, selecting different measurement intervals can well lead to varying results, and better understanding the measurand helps in selecting the most appropriate measurement interval for the application and in the interpreting of the results. To illustrate this, Figure 5 shows an example of waveform averages calculated over a sliding window, applied with different measurement time intervals and varying synchronization with the measured signal. Generally, an instrument based on the averaging algorithm produces only one value per assessed measurement time interval. Thus the output of the instrument contains equidistant samples taken from the curves shown in Figure 5.

Figure 5 shows how the averaging algorithm calculates the RMS current values of a 50 Hz symmetrical current waveform starting at 0 A, rising to a peak current of 1,41 A (RMS current 1 A) and then falling to a peak value of 1,13 A (RMS value of 0,8 A). When the measurement interval, containing N samples, equals one half-cycle, the output of Formula (6) rises from zero to 1 A in a half-cycle. It also reduces from 1 A to 0,8 A in a half-cycle. Similarly, when Formula (6) is used to compute the RMS values over 1 cycle and 10 cycles, the rise and fall times are respectively 1 cycle and 10 cycles.

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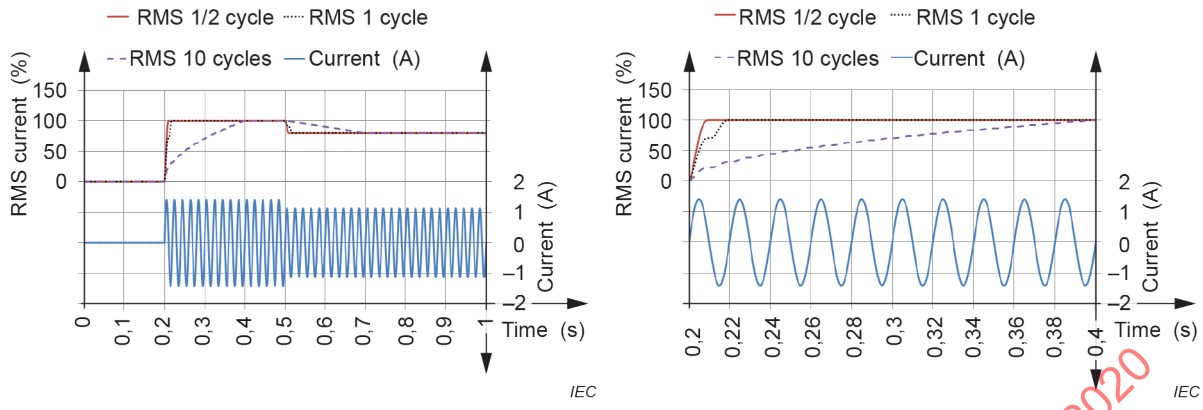

Whole test sequence **Detail**

Figure 5 – Step response of an algorithm in Formula (6) with a half-cycle, 1-cycle and 10-cycle measurement interval

As can be seen, averaging is very accurate for the determination of the RMS value of a periodic and sinusoidal waveform provided the samples contain an integer number of cycles. The results are available immediately upon completion of the measurement interval (10 ms for a half-cycle, or 200 ms for the 10-cycle average).

The measurement time interval equal to one half-cycle is used to analyse the fast fluctuation of voltages in the context of the directly measured parameters d_c, $d(t)$, d_{max} and T_{max} when performing flicker assessment. For this reason IEC 61000-4-15 specifies a half-cycle measurement between zero-crossings of the voltage.

In the extreme case, when the measurand is a half-wave rectified waveform so that a half-cycle measurement interval is mandatory, the synchronization of the measurement interval with the zero-crossing of the signal is very important, as the measurement interval is only half of the actual period of the current. If the measurement time interval is exactly synchronized to start and end at the zero-crossing of the voltage, the calculated values of the RMS current and active power alternate between 0 A and 1 A, or 0 W and 100 W, respectively, for the consecutive values calculated at the end of the measurement interval. The sliding average shown in Figure 6 gives these values at time instants indicated by circles.

In contrast, if the measurement time interval is synchronized to end at the positive and negative peaks of the voltage, then the RMS current and active power calculations produce constant values of 0,707 A and 50 W, respectively; this can be seen, for example, at time instants of 5 ms and 15 ms which are indicated by triangles in Figure 6. Between these two extreme cases of synchronisation, fluctuating values that are neither zero nor 1 A or 100 W are observed.

Therefore, when measuring signals based on half-wave rectification, a measurement interval equal to one or more full cycles should be used.

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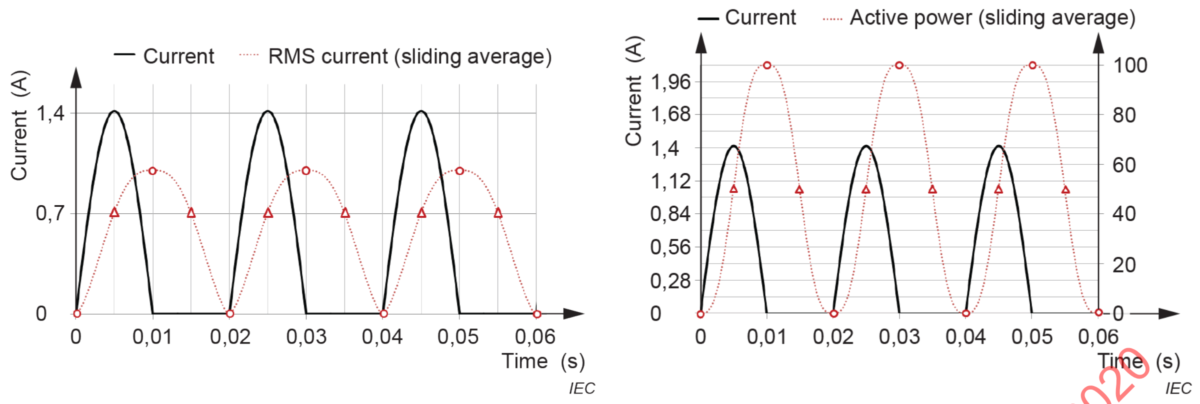

NOTE The voltage and currents are measured using half-cycle measurement intervals synchronized with the voltage zero-crossings (circles) and voltage peaks (triangles).

Figure 6 – RMS current and active power for half-wave rectification

The selection of the measurement time interval becomes even more critical for assessing the power quantities of equipment controlled by an MCSC circuit, a common technique to control devices with a heating element such as water heaters. As shown in 5.4, the 1/3 and 2/3 MCSC patterns have a period of three cycles of the fundamental mains frequency. Therefore, the energy consumed over one cycle is not constant. Selecting a measurement interval of one cycle would show this, and selecting a multiple of three cycles would give a constant value corresponding to the average power of the device over the MCSC period. MCSC devices can utilise different patterns within one device. For example, for a ceramic hob with a six-position control, the control period can be longer than 3 cycles. When the selected measurement time interval is not equal to one or more of the control periods of the current, fluctuations in the obtained values are observed. For example, the values obtained using a 10-cycle interval to assess pure 1/3 or 2/3 patterns can have about 15 % peak-to-peak variation; see Figure 7 and Figure 8.

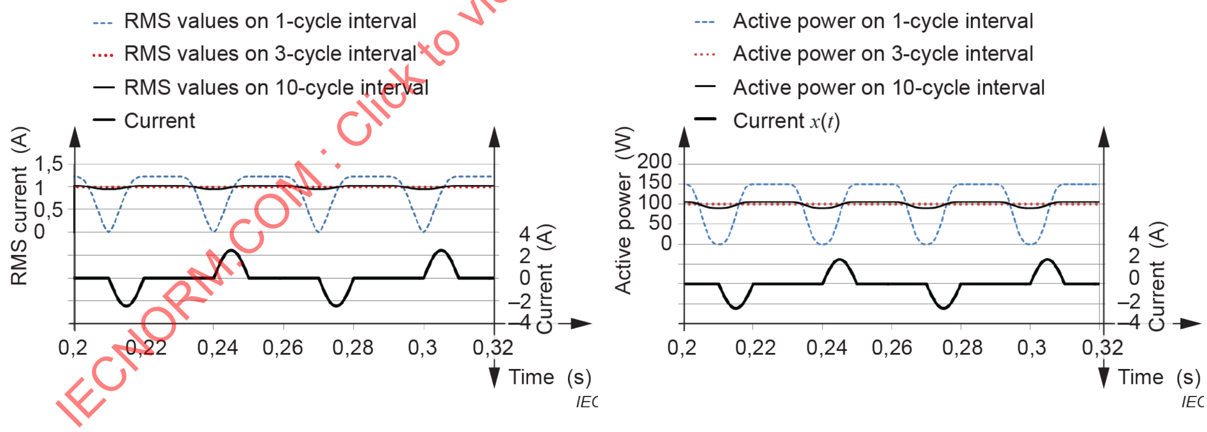

Figure 7 – Sliding average RMS current and active power of a device controlled with a 1/3 MCSC circuit

NOTE Figure 7 shows the different results for 1-, 3- and 10-cycle measurement intervals.

If all consecutive results from the sliding 10-cycle average measurements are smoothed with a 1,5 s low-pass filter, the peak variation of the filtered active power estimate is less than 0,03 % from the nominal value of 100 W. IEC 61000-4-7 recommends that the signal to be filtered is not the sliding average that is calculated after measuring every new sample but the average value updated after each measurement time interval, that is, one measurement per ten periods of 50 Hz or twelve periods of 60 Hz. When these values, having a 200 ms averaging interval, are filtered by the 1,5 s low-pass filter, variation up to 1 % is possible as shown in Figure 8. Therefore, the sliding average with a 1,5 s smoothing filter is more appropriate for the measurements of MCSC waveforms.

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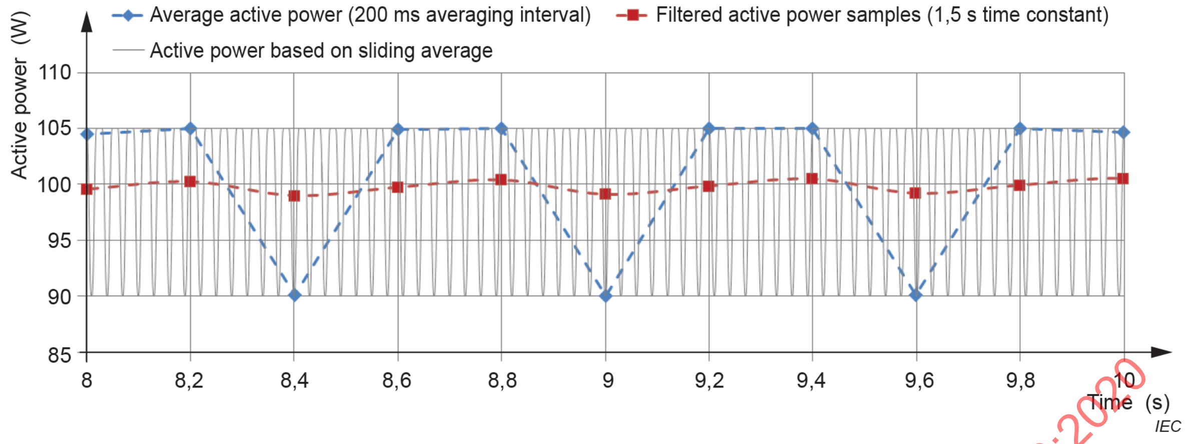

NOTE This shows the result when a 10-cycle interval average is calculated every 200 ms and subsequently digitally filtered using a low-pass filter with a 1,5 s time constant.

Figure 8 – Worst case 1/3 MCSC circuit active power calculation variation

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The situation with the 2/3 MCSC is similar to that with 1/3 MCSC but the variations with a 10-cycle measurement interval are about half of those in the 1/3 MCSC case. Table 1 shows the calculated power values for consecutive measurement windows for the 2/3 MCSC in a 50 Hz system. As the name implies, the control turns the power "on" for 2 out of every 3 half-cycles. So, for a 30 ms measurement window, consecutive windows show the exact same value. For non-multiples of 30 ms, consecutive measurement windows show different values, with the differences decreasing as the measurement window becomes longer. When averaged over approximately 10 s, the calculated power value shows negligible differences versus the theoretical average power, irrespective of the measurement window. For the 200 ms measurement window in accordance with IEC 61000-4-7, the power over two consecutive measurement windows differs by 7,692 %, simply because one 200 ms window includes 14 half-cycles that are "on" while the next one includes 13 half-cycles that are "on". Much like with the 1/3 MCSC, the 1,5 s smoothing filter would largely remove the differences, approximating the power over the MCSC period.

Table 1 – Calculated power of 2/3 MCSC for different measurement windows

Power calculation for 2/3 MCSC pattern with nominal 100 W (100 % "on") power					
Measurement window time T (ms)	**Minimum power (W)**	**Maximum power (W)**	**Average of two consecutive windows (W)**	**Delta (difference %) between consecutive windows**	**Average of all time windows over 10 s (W)**
30	666,667	666,667	666,667	0,000	666,667
50	600,000	800,000	700,000	33,333	666,700
60	666,667	666,667	666,667	0,000	666,667
100	600,000	700,000	650,000	16,666 7	666,700
180	666,667	666,667	666,667	0,000	666,667
200	650,000	700,000	675,000	7,692	666,675
300	666,667	666,667	666,667	0,000	666,667
1 000	660,000	670,000	665,000	1,515	666,670
3 000	666,667	666,667	666,667	0,000	666,667

All examples presented so far assume that a constant period of the signal exists, allowing the selection of the measurement time interval to be exactly one or more control periods to assess accurately the values of power quantities by averaging the samples. When the control circuit randomly switches the load on and off, no particular period can be identified. One approach is to calculate a 10/12-cycle interval average and process the obtained power quantity values using the 1,5 s low-pass filter as recommended in IEC 61000-4-7.

When the operation sequence of the device contains several states, the value measured with the averaging algorithm in 10/12-cycle intervals simply follows the variations illustrated in Figure 5 with a time constant of about 83 ms. If the goal is to estimate short-term values, then the 1,5 s filtered or a non-filtered measurement can be suitable. However, this time constant is not sufficient to average the effect of equipment that has a longer time constant. In this case, a longer measurement time interval is required. In the context of EMC, IEC documents propose to use a time interval between 10 min and 2 h, depending on the relevant type of equipment. For example, refrigerators with a variable speed drive, tested according to IEC 61000-3-2, require an observation time of 1 h. Whilst this is done mainly to accumulate sufficient statistical data on the harmonic emissions, longer time intervals also help in averaging the power.

Consider a current continuously fluctuating with a period of 92 s. The 10 min interval would contain about 6,5 of these periods. In this situation, the average of the samples contained in such 10 min intervals varies within ±7,3 % of the average value over the 92 s period, depending on the synchronization of the measuring interval (see Figure 9).

If the period of the RMS current fluctuates randomly between 60 s and 120 s, as shown in Figure 4, the 10 min measurement time interval can include fractional fluctuating periods. Subsequently, the measured active power values produced by this fluctuating current are likely to also fluctuate up to about ±10 % of the average value calculated over the whole 120 min interval (see Figure 10). Thus, in the worst case, two 10 min measurements could differ by 20 %.

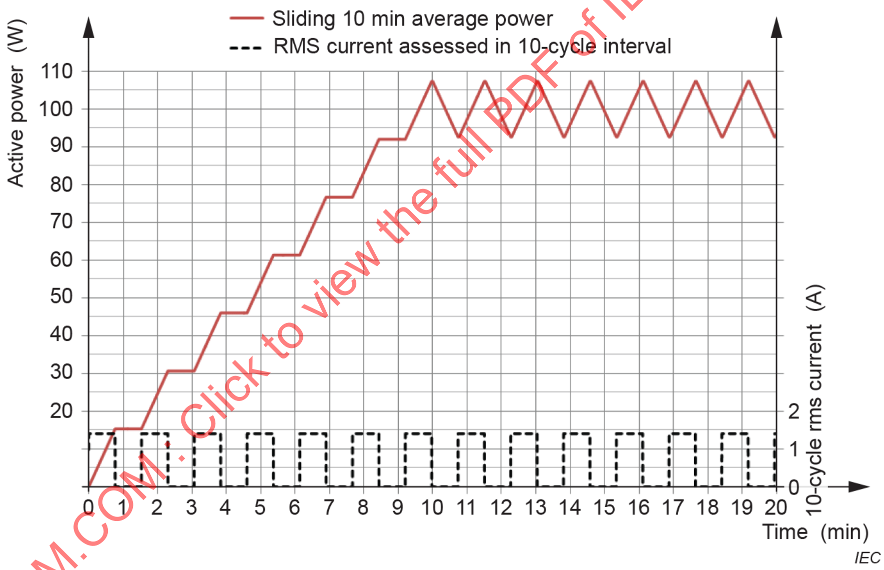

Figure 9 – Example of a 10 min sliding average power calculation for a load having a 92 s period

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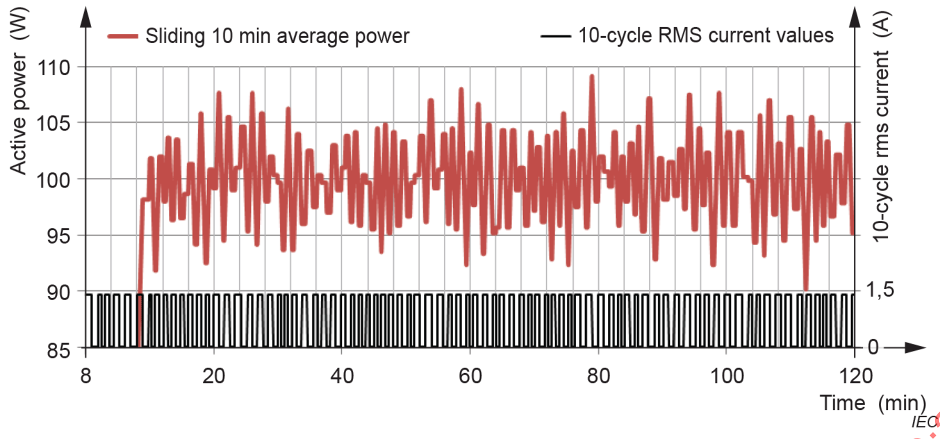

Figure 10 – Active power of randomly fluctuating load averaged over a sliding 10 min interval

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To assess the power produced by the fluctuating current shown in Figure 4, the measurement time interval for the averaging algorithm needs to exceed 15 min to ensure that the measured active power variation is within 5 % of the average value calculated over the whole 120 min interval.

6.2.3 Instrumental errors of the averaging algorithm

In all examples so far, the frequency of the power supply voltage was assumed to be nominal and the measurement time interval was selected according to this value (i.e. negligible instrumental errors). In practice the period of the mains frequency varies slightly over time. According to IEC 61000-4-7, the total measurement time interval for assessing harmonics needs to be matched to the period of the power system with an error of less than 0,03 %, but such tolerance is not required for power quantity assessment. For that reason, this document verifies the impact of imperfect synchronization when assessing power quantities.

When the sampling frequency is constant, the number of samples in the averaging is, of necessity, adjusted in order to obtain the precise measurement interval. This process can require a high sampling frequency to provide sufficient resolution to keep the interval duration error below 0,03 %. For example, a sampling frequency of 200 kHz, giving one sample per 5 µs, is adequate to assess an interval of 16,667 ms with an error less than 0,03 % (5 µs/16 667 µs = 0,03 %). When power quantities are assessed over longer intervals such as 200 ms, the sampling frequency can be reduced. This allows up to twelve missing samples for a sampling frequency of 200 kHz or one missing sample for a sampling frequency of 16,667 kHz.

Zero-crossings of the signal are often used to synchronize the samples in the searched interval. Other, more complicated algorithms such as a best fit of a sinusoidal function, are also common. The zero-crossing technique works well for a pure sine wave. Distortion in the signal, particularly with fluctuating amplitudes, alters the timing of the zero-crossings, which prevents finding the exact location for the measurement time interval. For this reason, filters can be used to reduce distortion prior to finding the zero-crossing. Unfortunately, these filters also add delays. A zero crossing found after the filtering step gives a correct duration of the measurement time interval but the start time of this interval is no longer correct due to the delay. Consequently, the start of the measurement interval is not necessarily synchronized with the real zero-crossing of the fundamental frequency signal. In some cases, this delay can affect the assessment of power quantities, particularly with half-wave rectification as shown earlier.

Another approach is to use a phase-locked loop (PLL) multiplier to maintain a constant number of samples in each measurement interval and adjust the sampling frequency to match a whole number of periods. The PLL multiplier multiplies the incoming fundamental frequency by a given ratio, and allows synchronous sampling, although the synchronicity is not necessarily guaranteed on a cycle-by-cycle basis. In real life situations, when large distortions or phase-shift jumps take place during network disturbances such as short circuits, even a perfectly designed PLL can sometimes fail to ensure synchronization. In the worst case the PLL can "lose lock", thereby causing the obtained data to be erroneous until synchronization is successfully restored.

In the case of the full-wave rectifier current waveform shown in Figure 11, the measured values of the RMS current and active power are 1 A and 100 W, respectively, when the measurement time interval is exactly equal to one or multiple cycles; see the case of 3 329 samples shown. However, the error in the assessment of power quantities increases when the time interval differs from the period of the assessed signal. This error is more pronounced in the assessment of active power. If the measurement interval of the instrument is within 0,03 % of the signal period, the measurement error generally remains below 0,15 %. When the measurement is made over several cycles, this error becomes smaller as the absolute number of unwanted samples remains constant, and the average is taken over a larger number of samples.

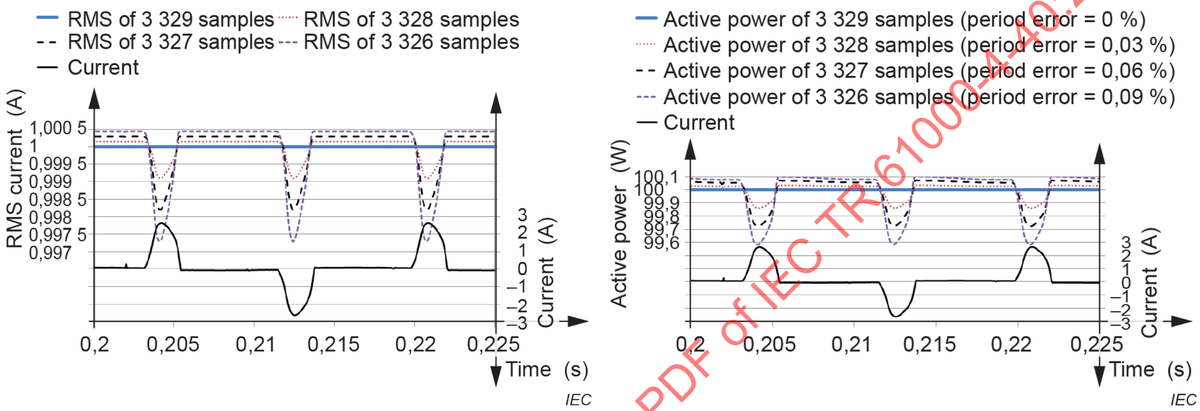

Figure 11 – Sensitivity of the full-bridge rectifier RMS current and active power measurement to time interval error of single-cycle sliding average calculation

6.3 Smoothing filter algorithm

6.3.1 Frequency and step response

The smoothing filter algorithm differs from the averaging algorithm by its filtering stage. Instead of calculating the average of the obtained samples, the filter algorithm aims to decrease the variation of the measurement results by applying a digital low-pass filter to each consecutive sample, smoothing them similarly to the mechanical damper in an analogue meter. While a well-designed smoothing algorithm, adapted to the application, gives practically the same value for a constant periodic signal as the averaging method, there are significant differences when the signal is not constant or periodic. However, the smoothing filter is not suitable for measuring rapid fluctuations in the observed signal.

The traditional moving-iron meter requires several seconds before the reading settles. For easy visual reading of the output of the meter, a damper with a time constant of about 1,5 s is sufficient. As shown in Figure 12, it takes 10 s for a first order filter with a 1,5 s time constant to reach 99,88 % of the step value. Further, about 0,1 % peak-to-peak ripple due to the high-frequency components of the squared sinusoid is still present in the output of the filter.

Modern instrumentation, implementing higher-order digital filter algorithms, makes it possible to achieve a stable output within a few cycles and substantially attenuate the unwanted frequency components, as demonstrated by the time-response curve of a 10th order filter shown in Figure 12.

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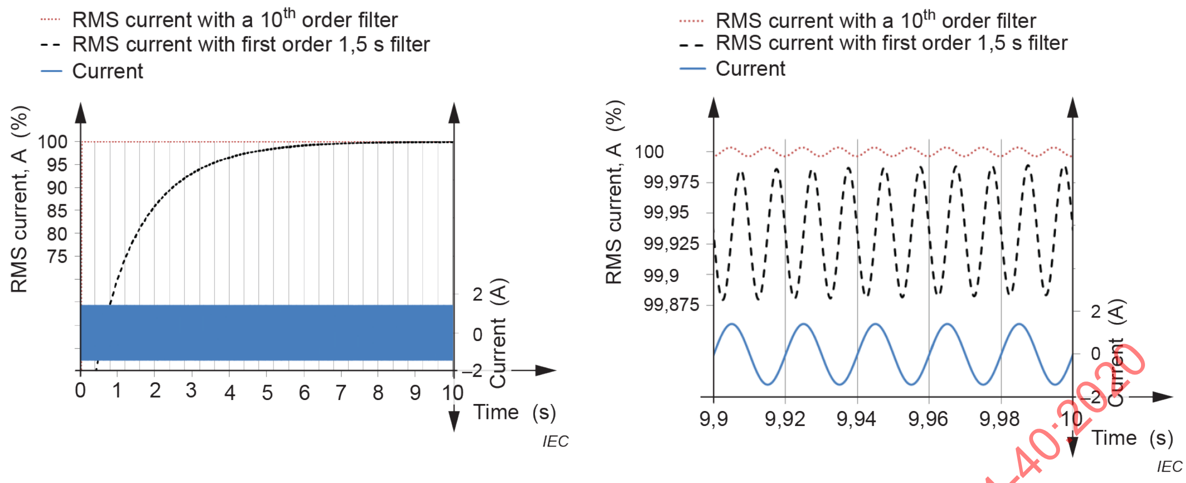

Step response **Detail**

Figure 12 – Comparison of the first and the 10th order filters used to estimate RMS current of a step signal

A properly designed low-pass filter has sufficiently low attenuation of the signal below the cut-off frequency, whilst above this frequency the attenuation rapidly increases. One frequently-used type of low-pass filter is the Butterworth filter. The frequency response of a 10th order Butterworth filter with the −3 dB cut-off frequency set to 10 Hz is shown in Figure 13.

As can be seen from Figure 13, the 10th order Butterworth filter has very rapid attenuation beyond its cut-off frequency. However, the step response of a Butterworth filter contains oscillations, as shown in Figure 14 for a Butterworth filter with a cut-off frequency of 60 Hz.

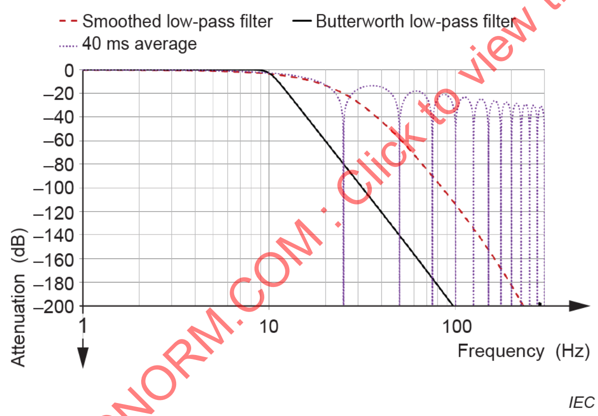

Figure 13 – Filter frequency responses

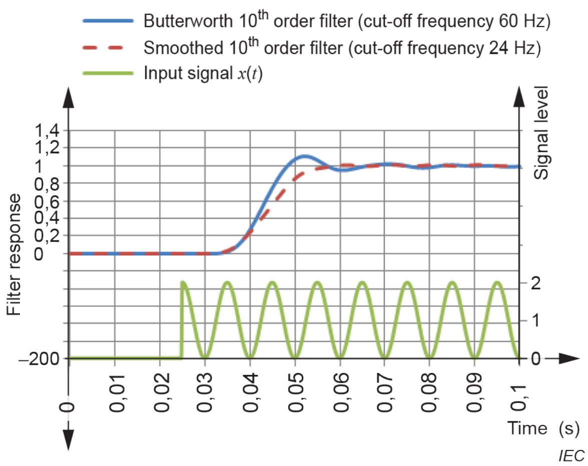

Figure 14 – Filter step responses

To obtain a step response that is more suitable for smoothing fluctuating signals, a filter with a smaller overshoot in the step response is needed. The frequency and step responses of one such filter, having the same cut-off frequencies as the Butterworth equivalent, are also presented in Figure 13 and Figure 14, respectively. This 10th order smoothing filter is studied further in this document and its detailed description is given in Annex A.

The cut-off frequency of a smoothing filter has a similar role to the measurement time interval in the averaging algorithm. In order to get fast settling and stable readings, the cut-off frequency is selected according to the period of the measured signal.

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However, the selection of the cut-off frequency is not as critical as the selection of the measurement interval. This can be seen from Figure 13, where the frequency response of an averaging algorithm with a 40 ms measurement time, which has roughly the same cut-off frequency at −3 dB, is also shown. Although the averaging has excellent attenuation for signals with a period equal to an integer fraction of the measurement interval, the attenuation is quite poor between these frequencies. On the contrary, the attenuation of the low-pass filters is high for all high frequencies. Thus low-pass filters can give stable readings for both fluctuating and unsynchronized signals. However, in doing so, the filter method deviates from the definitions in Formulae (1) to (3) in that the integration time is not a discrete period. Therefore, the filter algorithm can only give an approximation of the measured power quantity, albeit the uncertainty of this approximation can sometimes be low. Furthermore, in the case of fluctuating and unsynchronized signals, the filtering is only effective in assessing the average values of these signals and not the fluctuation itself. This is examined in detail in 6.3.2.

As a guide for selecting the cut-off frequency f_c of the smoothing filter under consideration, the following formula has been used, where T is the period of the measured signal:

$$f_c \leq \frac{1}{3T} \qquad (8)$$

Thus, for example, in the case of sinusoidal 50 Hz signals the cut-off frequency for RMS value and active power determination should be less than or equal to 16,667 Hz. For some applications, very different values of cut-off frequency are required. For example, some of the results presented further in 6.3.2 were obtained with a much lower cut-off frequency, such as 0,001 67 Hz in Figure 21. Therefore, as in the case of the averaging algorithm, knowledge of the measurand helps in selecting the most appropriate filter parameters and in interpreting the results.

6.3.2 Verification of the smoothing filter algorithm

With the 50 Hz sinusoidal signal shown in Figure 15 the active power calculation smoothed with a 10th order filter described in Annex A, designed for f_s = 100 kHz sampling frequency and 16,667 Hz cut-off frequency, follows closely the amplitude change in the current. The step response of this filter has about a one-period delay time and about a two-period response time to 90 % of the steady state value, as illustrated in Figure 16. The settling time to 99,95 % of the steady state value is about three periods.

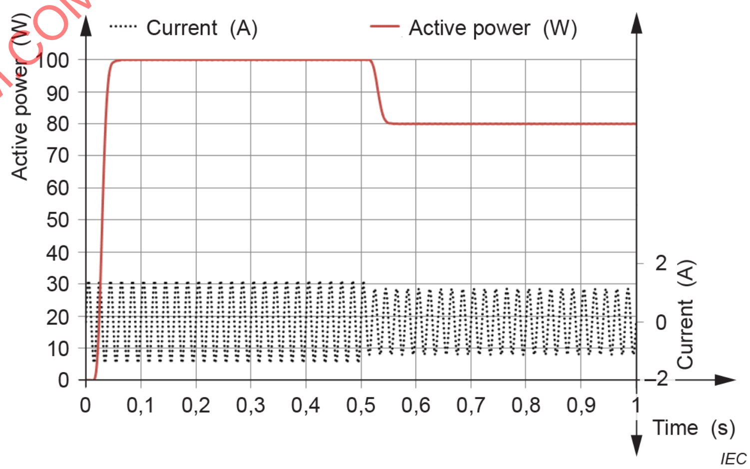

Figure 15 – Output of the 10th order smoothing filter used to calculate the active power of a signal with a step change

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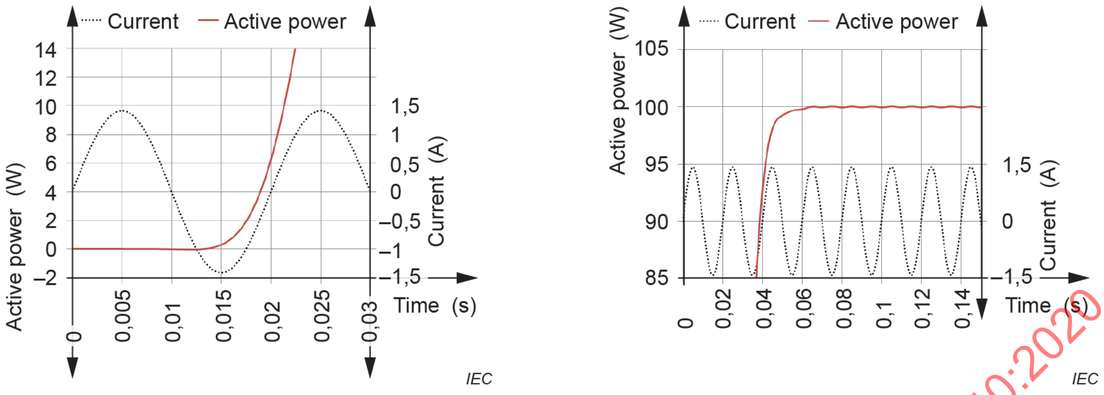

Figure 16 – Delay and response time of a 10th order filter used to assess the sinusoidal current of a sinusoidal waveform

When compared with the results of the averaging algorithm shown in Figure 5, it is clear that with perfect sinusoidal signals the averaging algorithm is able to give exact results much faster than the smoothing filter, as the averaging time can be set as low as half the period of the current without violating the power quantity definitions. Of course, if the averaging period is set to 10 cycles, such as is the case for the 200 ms measurement interval in IEC 61000-4-7, the averaging is significantly slower than the filtering.

Further, even though the 10th order smoothing filter attenuates the ripple of the instantaneous power considerably, it still leaves a frequency component with about 0,025 % amplitude for the case shown in Figure 16. The averaging would, theoretically, remove all fluctuations.

The above rule to choose the cut-off frequency to be the inverse of three periods assumes a symmetric waveform that, once squared, has twice the frequency of the input signal. The squaring of the half-wave rectified current does not double the frequency. Consequently, for the half-wave rectification case the cut-off frequency should correspond to at least five basic periods. Thus for a 50 Hz power system the cut-off frequency could be less than or equal to 10 Hz. As shown in Figure 17, the active power values obtained using the filtering algorithm with the 10 Hz cut-off frequency oscillate around the average value with an amplitude of ±0,06 %.

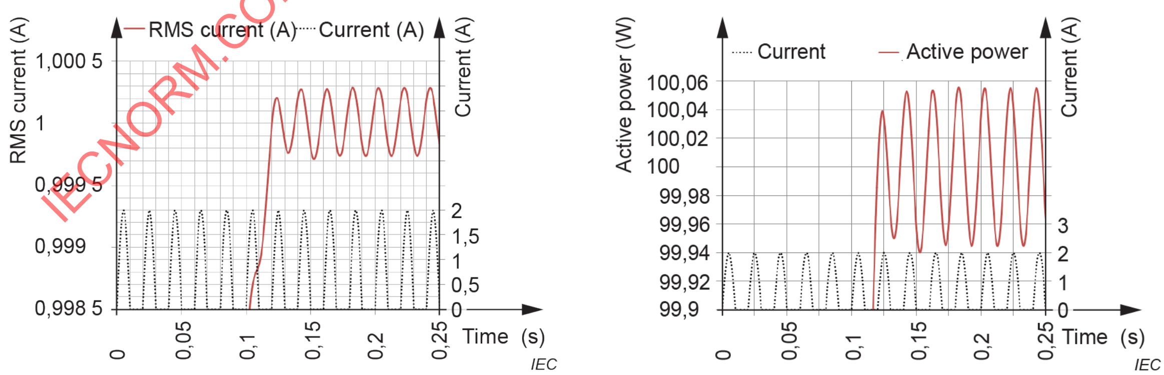

Figure 17 – Measurement of the current and power of a half-wave rectified signal using a smoothing filter with a 10 Hz cut-off frequency

In case of full-wave rectification, the power quantities can be successfully assessed using a 16,667 Hz cut-off frequency. The remaining unfiltered signal gives an oscillation up to ±0,05 % of the average power over one cycle, as shown in Figure 18. Since the filter needs several time constants to settle, the filter algorithm is not ideal for assessing signals that vary within the cycle.

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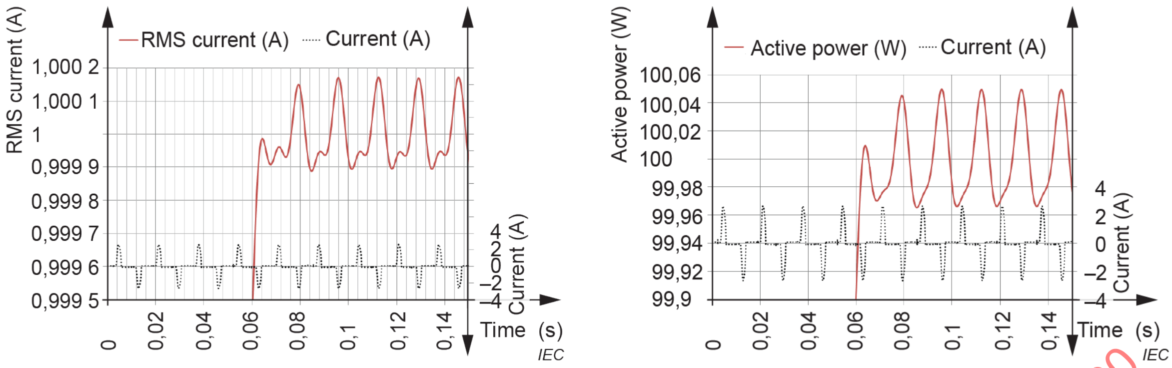

Figure 18 – Power quantities in full wave rectification assessed using a smoothing filter with 16,667 Hz cut-off frequency

The MCSC waveform is similar to the full-wave rectified waveform except that the period is three times longer. The period of the current is 60 ms, thus requiring a cut-off frequency of approximately 5,556 Hz. When selecting this frequency, the measured values of active power remain within ±0,05 % of the average (see Figure 19). It is important to note that this level of accuracy can be reached regardless of the variation in the mains frequency and without the need to synchronize the measurement. By slightly decreasing the cut-off frequency to 5 Hz, the fluctuation of the filtered active power result can be further reduced to about ±0,02 % but at the expense of a slightly longer delay.

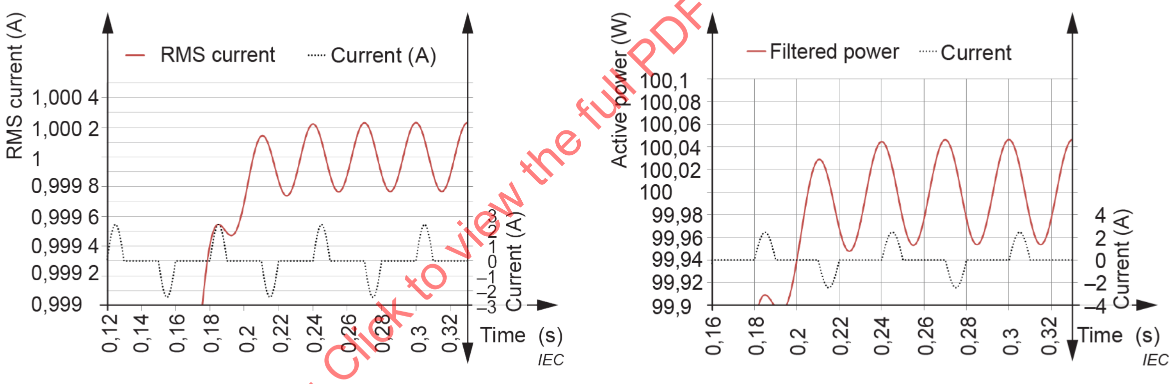

Figure 19 – MCSC 1/3 pattern power quantities filtered with approximately 5,556 Hz cut-off frequency

As was the case with the 10-cycle averaging algorithm, the deviation produced by a smoothing filter also reduces by half when the MCSC has the 2/3 pattern.

Power quantities with random fluctuating periods (see Figure 4) can be assessed using, for example, a 5 Hz cut-off frequency that tracks the RMS current and the active power roughly the same as a sliding average of 10 cycles.

For signals with very long periods, the cut-off frequency can be decreased accordingly. Figure 20 shows an example of the fluctuating power of the 92 s periodic load, shown earlier in Figure 9, smoothed with a 0,001 67 Hz cut-off filter. As can be seen, the response time of the smoothing filter is similar to that of the 10 min sliding average. However, the variation of the power calculated by the smoothing filter is only ±0,015 W. For comparison, the worst case difference in the successive average values of power, computed for each 10 min interval, can be as high as about 13 W. At the same time, the average power values of the same signal calculated over a 30 min period is constant.

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It is important to note that although the averaging algorithm with a 92 s measuring time interval would give an exact average value without any fluctuations, the increase of the time interval to 10 min (600 s) still gives fluctuating results in spite of the approximately six times longer averaging time. Thus, if the period of the load variation is not accurately known, which is rather common, it is usually not practical to increase the averaging time interval to such extent that it reduces the variation of the measured results. Minimizing the fluctuations by finding an optimum measuring time interval by trial and error is usually tedious. With the smoothing/filtering algorithm demonstrated here, decreasing the cut-off frequency always reduces fluctuations and no accurate knowledge of the period is needed.

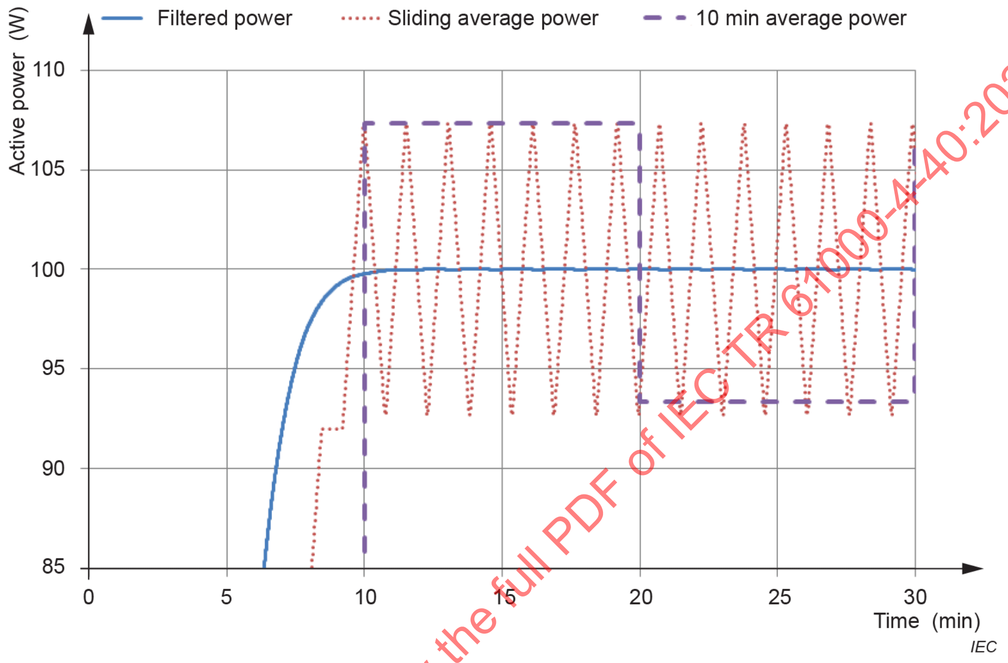

Figure 20 – Active power of a load having a 92 s period measured with different algorithms

Figure 21 shows the fluctuating signal of Figure 4, with periods randomly varying between 1 min and 2 min, smoothed with an 0,001 67 Hz cut-off filter. For comparison the sliding 10 min average value from Figure 10 is also shown. As can be seen, the peak-to-peak fluctuations are approximately halved.

To reach a more stable value, the cut-off frequency can be decreased further. Thus, in case of a randomly varying load it is quite straightforward to select such a cut-off frequency that, for example, preserves low frequency load variation but filters away the higher frequencies caused by random on/off control. With the averaging algorithm, the measured low-frequency load changes are significantly influenced by the random higher frequencies when they are, as highlighted by Figure 13, not located close to frequencies that are the inverse of the measurement time interval or a multiple of it.

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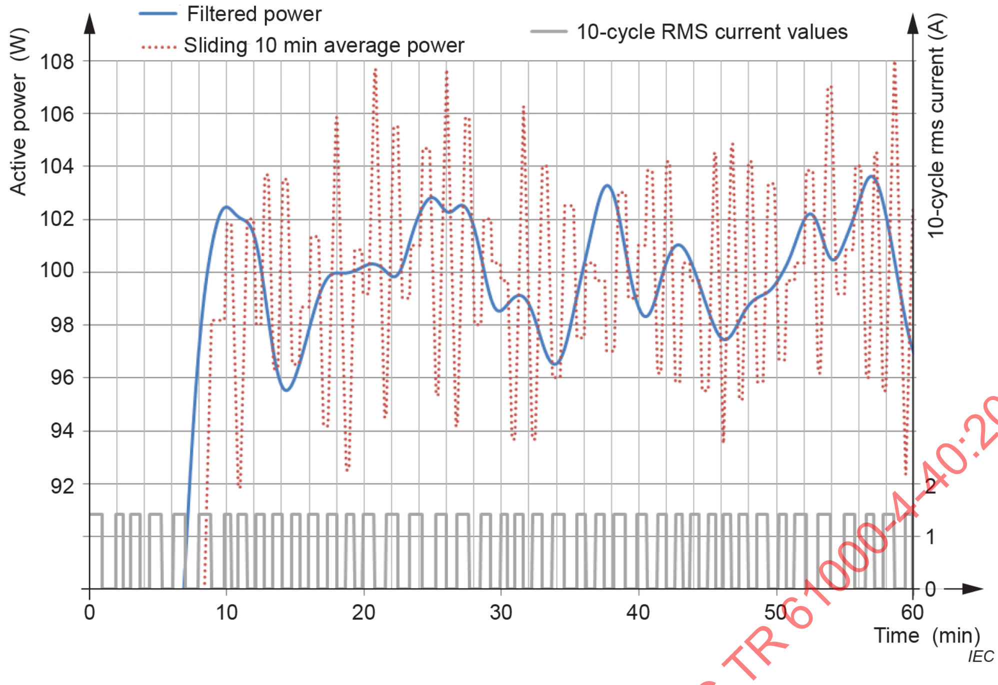

Figure 21 – Active power of randomly fluctuating load measured using different algorithms

6.3.3 Instrumental errors of the filtering algorithm

As in the case of the averaging algorithm, the implementation of the filter algorithm can also give rise to instrumental errors. The parameters of the filter depend critically on the precision of the calculation. If the size of the digital word is too small for the required precision, the errors due to rounding can become significant. They can result in slight changes in the cut-off frequency or, more importantly, in a wrong value of the DC component, which directly affects the result.

These effects happen, particularly, when there is a high ratio between the sampling frequency and the cut-off frequency. In the filter described here this ratio can reach 100 kHz/0,001 Hz = 1 × 10^8.

Therefore, to obtain the maximum benefits of the filtering algorithm, its practical realization can require significant hardware investment, such as an advanced processor with 80-bit floating numbers. Simpler devices with a 32-bit floating arithmetic or, worse still, an integer or fixed point arithmetic can cause dramatic instrumental errors. Annex A addresses this issue in more detail.

7 Conclusions

The averaging algorithm measures the power quantities according to the definitions in Clause 4 and always returns the value of the average power over the chosen measurement period. It is particularly useful when the measured signals are perfectly periodic, such as the voltages and currents usually measured in 50 Hz and 60 Hz power systems and when the measurement window includes one or more integer cycles of the fluctuating power. With such signals the averaging method is able to give exact results and the averaging time, i.e. the measurement window, can be set as low as half the period of the measured current and voltage without violating the power quantity definitions.

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In practice the measured electrical power quantities often contain distortion and load variations that are not synchronized with the power system frequency, thus they tend to cause fluctuations in the measured values. For that reason, many applications, such as those for harmonic emissions in accordance with IEC 61000-4-7, require attenuating these variations and specify additional low-pass filtering. The drawback of the additional low-pass filtering is the long response time due to the time constant of the filter.

The higher-order smoothing filter described in this document does not need to be synchronized to the power supply frequency, but, for best performance its cut-off frequency is adapted to the modulation frequency and other parameters of the measured signal, as highlighted by Figure 20. The filter can attenuate the variation in the measured signal to almost zero and achieve a response time that is similar to the time used for the average calculation. However, the smoothing filter is not suitable for measuring rapid signal variations. Strictly speaking, it does not comply with Formulae (1) to (3) of Clause 4 because it does not allow the assessment of power quantities over a given measurement interval. Nevertheless, even without the exact knowledge of the period, the method can produce an accurate estimate of the power quantity.

The implementation of the averaging according to Formulae (4) to (7), requires the averaging to match closely one or more cycles of the signal frequency as defined in IEC TR 61000-1-7. Consequently, the measurement instrument needs to accurately assess the mains frequency and continuously adjust the sampling frequency or the number of samples to obtain the appropriate measurement time interval. Conversely, the implementation of the filter method requires more complex and, generally, more precise calculations. For example, the smoothing filter algorithm presented takes about 20 times more memory and requires the calculation of about 40 times more arithmetic operations than the calculation of the average.

Although the proposed smoothing filter algorithm is more complicated to implement than the calculation of the average, it does not require exact knowledge of the signal period and thus does not need phase locked loops or other means to synchronize with the measured signals. Therefore, the higher order smoothing filter method can be used to assess signals with randomly fluctuating signals. Generally, the filter method is appropriate and helpful, when the cut-off frequency of the filter can be specified for the actual application. If the switching frequency of the load or the disturbance frequency is three or more times higher than the cut-off frequency of the filter, reasonable results can be expected.

To achieve optimal results, the selection of both the measurement interval for the averaging measurement and the time constant of the filter depends greatly on the parameters of the signal to be measured.

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Annex A
(informative)

Smoothing filter studied in this document

A.1 Algorithm

Several low-pass filter types can be used for attenuating the AC components and obtaining the DC level of a signal. The main criterion for the measurement of power quantities is the ability of the filter to respond without oscillation to an amplitude step from 0 % to 100 % of a squared sinusoidal signal. Annex A gives one version of possible filter implementations, however other versions can also perform well.

The smoothing filter algorithm can be implemented in the firmware of measuring instruments, user-programmable measuring systems, or, when used to post-process measured data, in a computer program or spreadsheet calculation. The examples shown in this document were produced by spreadsheet calculations.

The proposed 10th order low-pass filter with additional damping represents an infinite impulse response type (IIR) filter. The version presented takes advantage of cascaded second-order stages to compose a multi-order filter. This type of filter is very popular in real-time processing applications as it reduces the computing resources required, particularly for demanding low-frequency cut-off requirements. The filter coefficients are calculated with the cut-off frequency f_c and the sampling frequency f_s as input parameters.

Formulae (A.1) to (A.5) give the formulae for the cascaded stages filtering the input signal $x(t)$. This signal can be the squared value of the voltage or the current or the product of each sample of voltage and current. The same formula is used for each second order stage, with the output of each stage being the input for the next stage. The last formula describes the filtered output signal $X(t)$. When the input is the squared voltage or current, the square root of the output $X(t)$ gives the RMS voltage or RMS current. When the input is the product of current and voltage, the output $X(t)$ directly gives the active power.

This recursive process computes the output $X(t)$ for each sampling interval Δt. The process starts with all $W(t)$ values equal to zero. At each step, each consecutive formula re-uses the previous values of $W(t)$ to compute the filtered signal $X(t)$.

The formulae are

$$W_1(t) = A_1\left[x(t) + 2x(t-\Delta t) + x(t-2\Delta t)\right] - \left(B_1 W_1(t-\Delta t) + C_1 W_1(t-2\Delta t)\right) \quad \text{(A.1)}$$

$$W_2(t) = A_2\left[W_1(t) + 2W_1(t-\Delta t) + W_1(t-2\Delta t)\right] - \left(B_2 W_2(t-\Delta t) + C_2 W_2(t-2\Delta t)\right) \quad \text{(A.2)}$$

$$W_3(t) = A_3\left[W_2(t) + 2W_2(t-\Delta t) + W_2(t-2\Delta t)\right] - \left(B_3 W_3(t-\Delta t) + C_3 W_3(t-2\Delta t)\right) \quad \text{(A.3)}$$

$$W_4(t) = A_4\left[W_3(t) + 2W_3(t-\Delta t) + W_3(t-2\Delta t)\right] - \left(B_4 W_4(t-\Delta t) + C_4 W_4(t-2\Delta t)\right) \quad \text{(A.4)}$$

$$X(t) = A_5\left[W_4(t) + 2W_4(t-\Delta t) + W_4(t-2\Delta t)\right] - \left(B_5 X(t-\Delta t) + C_5 X(t-2\Delta t)\right) \quad \text{(A.5)}$$

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where

Δt	= sampling time interval (s): $\Delta t = \frac{1}{f_s}$
$x(t)$	= input value at time t.
$x(t\text{-}\Delta t)$	= input value at time $t\text{-}\Delta t$
$x(t\text{-}2\Delta t)$	= input signal at time $t\text{-}2\Delta t$
$W_1(t)$, $W_2(t)$, $W_3(t)$, $W_4(t)$	= intermediate values at time t
$W_1(t\text{-}\Delta t)$, $W_2(t\text{-}\Delta t)$, $W_3(t\text{-}\Delta t)$, $W_4(t\text{-}\Delta t)$	= intermediate values for $W(t)$ at time $t\text{-}\Delta t$
$W_1(t\text{-}2\Delta t)$, $W_2(t\text{-}2\Delta t)$, $W_3(t\text{-}2\Delta t)$, $W_4(t\text{-}2\Delta t)$	= intermediate values for $W(t)$ at time $t\text{-}2\Delta t$
$X(t)$	= filtered value at time t
$X(t\text{-}\Delta t)$	= filtered value at time $t\text{-}\Delta t$
$X(t\text{-}2\Delta t)$	= filtered value at time $t\text{-}2\Delta t$

The coefficients of the algorithm are defined by the following

$$\psi = \tan\left(\frac{2\pi\sqrt{2}f_c}{f_s}\right) \tag{A.6}$$

$$D_i = \frac{1}{\psi^2 + 2\gamma\psi\sin\left(\frac{(13-2i)\pi}{20}\right) + 1} \tag{A.7}$$

$$A_i = \psi^2 D_i \tag{A.8}$$

$$B_i = 2\left(\psi^2 - 1\right) D_i \tag{A.9}$$

$$C_i = \left(\psi^2 - 2\gamma\psi\sin\left(\frac{(13-2i)\pi}{20}\right) + 1\right) D_i \tag{A.10}$$

where

i	=	running integer number between 1 and 5
γ	=	additional damping factor = 1,036
f_c	=	cut-off frequency (Hz)
f_s	=	sampling frequency (Hz)

As an example of the calculation of the coefficients for the Formulae (A.1) to (A.5), let us assume a 50 Hz signal sampled at 100 kHz. The fundamental signal period T is then 20 ms. In order to obtain sufficient attenuation, a cut-off frequency corresponding to at least three signal periods should be selected. Thus f_c should be less than 16,667 Hz. With a 16,667 Hz cut-off frequency, the filter parameters for a 100 kHz sampling frequency are computed as follows:

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