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Geometrical product specifications (GPS) — Filtration —

Part 29:
Linear profile filters: Spline wavelets

Spécification géométrique des produits (GPS) — Filtrage —

Partie 29: Filtres de profil linéaires: Ondelettes splines

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Foreword

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An ISO/PAS or ISO/TS is reviewed after three years in order to decide whether it will be confirmed for a further three years, revised to become an International Standard, or withdrawn. If the ISO/PAS or ISO/TS is confirmed, it is reviewed again after a further three years, at which time it must either be transformed into an International Standard or be withdrawn.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TS 16610-29 was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

ISO/TS 16610 consists of the following parts, under the general title *Geometrical product specifications (GPS) — Filtration*:

— *Part 1: Overview and basic concepts*

— *Part 20: Linear profile filters: Basic concepts*

— *Part 22: Linear profile filters: Spline filters*

— *Part 29: Linear profile filters: Spline wavelets*

— *Part 31: Robust profile filters: Gaussian regression filters*

— *Part 32: Robust profile filters: Spline filters*

— *Part 40: Morphological profile filters: Basic concepts*

— *Part 41: Morphological profile filters: Disk and horizontal line-segment filters*

— *Part 49: Morphological profile filters: Scale space techniques*

The following parts are under preparation:

— *Part 21: Linear profile filters: Gaussian filters*

— *Part 26: Linear profile filters: Filtration on nominally orthogonal grid planar data sets*

— *Part 27: Linear profile filters: Filtration on nominally orthogonal grid cylindrical data sets*

— *Part 30: Robust profile filters: Basic concepts*

— *Part 42: Morphological profile filters: Motif filters*

— *Part 60: Linear areal filters: Basic concepts*

— *Part 61: Linear areal filters: Gaussian filters*

— *Part 62: Linear areal filters: Spline filters*

— *Part 69: Linear areal filters: Spline wavelets*

— *Part 70: Robust areal filters: Basic concepts*

— *Part 71: Robust areal filters: Gaussian regression filters*

— *Part 72: Robust areal filters: Spline filters*

— *Part 80: Morphological areal filters: Basic concepts*

— *Part 81: Morphological areal filters: Sphere and horizontal planar segment filters*

— *Part 82: Morphological areal filters: Motif filters*

— *Part 89: Morphological areal filters: Scale space techniques*

Introduction

This part of ISO/TS 16610 is a geometrical product specification (GPS) Technical Specification and is to be regarded as a global GPS Technical Specification (see ISO/TR 14638). It influences the chain links 3 and 5 of all chains of standards

For more detailed information of the relation of this part of ISO/TS 16610 to the GPS matrix model, see Annex E.

This part of ISO/TS 16610 develops the terminology and concepts for spline wavelets.

Geometrical product specifications (GPS) — Filtration —

Part 29:
Linear profile filters: Spline wavelets

1 Scope

This part of ISO/TS 16610 specifies spline wavelets for profiles, and contains the relevant concepts. It gives the basic terminology for spline wavelets of compact support, together with their usage.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO/TS 16610-1:2006, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic terminology*

ISO/TS 16610-20:2006, *Geometrical product specifications (GPS) — Filtration — Part 20: Linear profile filters: Basic concepts*

ISO/TS 16610-22:2006, *Geometrical product specifications (GPS) — Filtration — Part 22: Linear profile filters: Spline filters*

International vocabulary of basic and general terms in metrology (VIM). BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 2nd ed., 1993

3 Terms and definitions

For the purposes of this document, the terms and definitions given in VIM, ISO/TS 16610-1, ISO/TS 16610-20 and ISO/TS 16610-22, and the following apply.

3.1
mother wavelet
function of one or more variables which forms the basic building block for wavelet analysis, related to a scalar function

NOTE A mother wavelet usually integrates to zero, is localized in space and has a finite bandwidth. Figure 1 provides an example of a real valued mother wavelet.

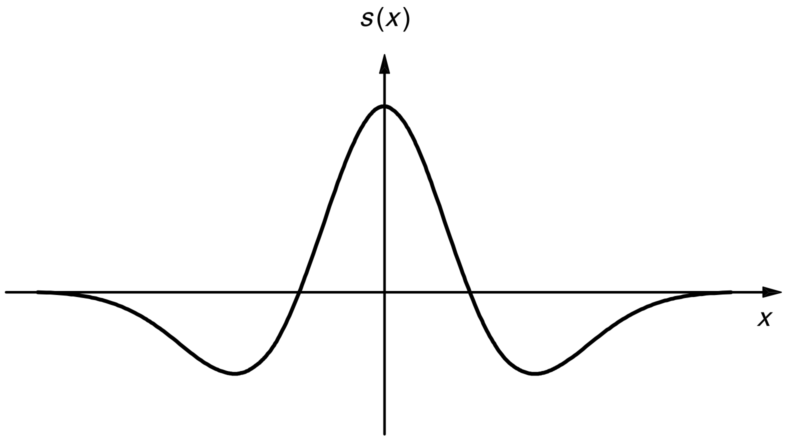


Figure 1 — Example of a real valued mother wavelet

3.2
wavelet family
$g_{a,b}$
family of functions generated from the **mother wavelet** (3.1) by dilation and translation

NOTE If $g(x)$ is the mother wavelet, then the wavelet family $g_{a,b}(x)$ is generated as follows:

$$g_{a,b}(x) = a^{-0,5} \times g\left(\frac{x-b}{a}\right) \tag{1}$$

where

a is the dilation parameter;

b is the translation parameter.

3.2.1
dilation
⟨wavelet⟩ transformation which scales the spatial variable x by a factor a

NOTE 1 This transformation takes the function $g(x)$ to $a^{-0,5}g(x/a)$ for an arbitrary positive real number a.

NOTE 2 The factor $a^{-0,5}$ keeps the area under the function constant.

3.2.2
translation
transformation which shifts the spatial position of a function by a real number b

NOTE This transformation takes the function $g(x)$ to $g(x - b)$ for an arbitrary real number b.

3.3
discrete wavelet transform
unique decomposition of a profile into a linear combination of a **wavelet family** (3.2) where the **translation** (3.2.2) parameters are integers and the **dilation** (3.2.1) parameters are powers of a fixed positive integer greater than 1.

NOTE 1 The dilation parameters are usually powers of 2.

NOTE 2 Throughout the rest of this part of ISO/TS 16610, the discrete wavelet transform is referred to as the wavelet transform.

3.4
multiresolution analysis
decomposition of a profile by a filter bank into portions of different scales

NOTE 1 The portions at different scales are also referred to as resolutions.

[ISO/TS 16610-20:2006]

NOTE 2 See Figure 2.

NOTE 3 Since there is, by definition, no loss of information, it is possible to reconstruct the original profile from the multiresolution ladder structure.

3.4.1
low-pass component
component obtained after convolution with a smoothing filter (low pass) and a decimation

3.4.2
high-pass component
component obtained after convolution with a difference filter (high pass) and a decimation

NOTE The weighting function of the difference filter is defined by the wavelet from a particular family of wavelets, with a particular dilation parameter and no translation.

3.4.3
multiresolution ladder structure
structure consisting of all the orders of the difference components and the highest order smooth component

3.4.4
scalar function
function which defines the weighting function of the smoothing filter used to obtain the smooth component

NOTE In order to avoid loss of information on the multiresolution ladder structure, the wavelet and scaling function are matched.

3.4.5
decimation
⟨wavelet⟩ action which samples every kth point in a sampled profile, where k is a positive integer

NOTE Typically k is equal to 2.

3.5
spline wavelet
wavelet family (3.2) whose corresponding reconstructing **scalar functions** (3.4.4) are splines

4 General wavelet description

4.1 General

A spline wavelet claiming to comply with this document shall satisfy the equations given in Annex A.

NOTE Examples of the application of cubic of interpolating spline wavelets are given in Annex B. A concept diagram for the concepts for spline wavelet filters is given in Annex C, and the relationship to the filtration matrix model is given in Annex D.

4.2 Basic usage of wavelets

Wavelet analysis consists of decomposing a profile into a linear combination of wavelets $g_{a,b}(x)$, all generated from a single mother wavelet (see [6]). This is similar to Fourier analysis, which decomposes a profile into a linear combination of sinewaves, but unlike Fourier analysis, wavelets can identify the location as well as the scale of a feature in a profile. As a result, they can decompose profiles where the small-scale structure in one portion of the profile is unrelated to the structure in a different portion, such as localized changes (i.e. scratches). Wavelets are also ideally suited for non-stationary profiles. Basically, wavelets decompose a profile into building blocks of constant shape, but of different scales.

4.3 Wavelet transform

The discrete wavelet transform of a profile $s(t)$ given at fixed intervals $x_i = i\Delta x$, (where Δx is the sampling interval and i = ...,-2,-1,0,1,2,...) with the mother wavelet $g(x)$ is given by

$$S\left(i\Delta x,a\right)=\Delta x\sum_{j}s\left[\left(i-j\right)\Delta x\right]g_{a,j\Delta x}\left(j\Delta x\right) \tag{2}$$

The dilation parameter a is also restricted to discrete values. Consecutive values of a usually have a fixed ratio, i.e. a_i / a_{i+1} = constant.

NOTE 1 This constant is usually 2.

If the wavelet $g(x)$ has a finite spatial extent, the number of sampling points of $t(x)$ at the scale a grows linearly with a, such that calculations of S with an algorithm based on the above equation are generally impractical. Efficient wavelet algorithms depend on various properties of the mother wavelet. The algorithm considered here is that for multiresolution (see ISO/TS 16610-20:2006), which is valid for “biorthogonal” wavelets, to which spline wavelets belong.

The multiresolution form of the wavelet transform consists of constructing a ladder of smooth approximations to the profile (see Figure 2). The first rung is the original profile. Each rung in the ladder consists of a filter bank (see ISO/TS 16610-20:2006), where the profile S^i is split into two components:

— a smoother version of the profile S^{i+1}, which becomes the next rung, and

— a component that is the “difference” between the two rungs d^{i+1}.

The action of the low pass (smoothing) filter H_0 and the high pass filter H_1 in the filter bank reduces the number of profile points by half.

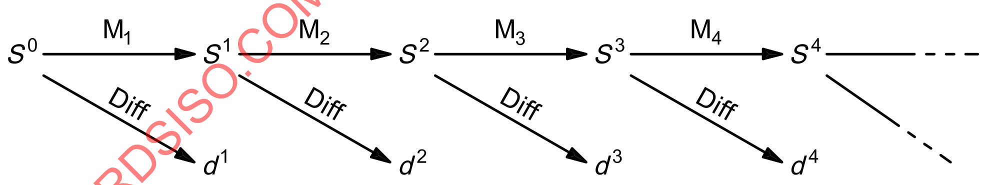


Figure 2 — Example of a multiresolution separation using a wavelet transform

The original profile can be reconstructed from (d^1, d^2, d^3, ... d^n, S^n) by reversing the ladder structure and using a second pair of filters H'_0 and H'_1. Wavelets do not consist of a single method (like the Fourier Transform), but of a multitude of transforms dependent on a mother wavelet, which determines the four filters, H_0, H_1, H^*_0 and H^*_1. Examples of wavelet transforms of profiles using multiresolution can be found in Annex B.

NOTE 2 There are other forms of splitting the profile using filter banks. The above is just one example.

4.4 Spline wavelets

Symmetrical wavelets include spline wavelets. Spline wavelets are families of wavelets whose corresponding scalar functions are splines.

NOTE 1 The second generation wavelet algorithm is an efficient method for computing wavelet transformations.

NOTE 2 All wavelets with a finite number of filter coefficients can be represented as second generation wavelets.

4.5 Nested mathematical models

The multiresolution ladder structure lends itself naturally to a set of nested mathematical models of the profile, with the ith model, m^i, reconstructed from (d^i, d^{i+1}, ... d^n, s^n), as illustrated in Figure 2. The order of the model is equivalent to a cut-off value: the higher the order of the model, the smoother the representation. Thus, m^{i+1} is a smoother version of the profile than m^i.

A quantity similar to the "transmission bandwidth" can be constructed using the nested mathematical models, by calculating the height difference between two specified profiles, e.g.

$$\mathrm{m}^{i,j} = \mathrm{m}^i - \mathrm{m}^j \qquad (3)$$

where $i < j$.

Thus, in this particular example, order i is equivalent to cut-off value λ_s and order j is equivalent to cut-off value λ_c The exact relationship between the model order and the cut-off value is dependent on the particular mother wavelet chosen.

5 Recommendations

5.1 Spline wavelet

If not otherwise specified, an interpolating cubic spline wavelet shall be used (see Annex A).

5.2 Nesting Index (cut-off values λ_c)

It is recommended that the nesting index (the cut-off value λ_c) be chosen from a logarithmic series (constant ratio) of values. Experience has shown that a constant ratio of approximately the square root of ten between successive scale values is optimal. The nesting index should be chosen from the following series of values:

... 2,5 µm; 8 µm; 25 µm; 80 µm; 250 µm; 0,8 mm; 2,5 mm; 8 mm; 25 mm; ...

6 Filter designation

Spline filters in conformance with this part of ISO/TS 16610 are designated

FPLW

See also ISO/TS 16610-1:2006, Clause 5.

Annex A
(normative)

The family of interpolating spline wavelets

A.1 General

The lifting scheme is used to define the family of interpolating spline wavelets (see [3][4][5]). Starting with the original sampled profile, each rung in the multiresolution ladder is calculated from the previous rung in three stages. These stages are called:

— splitting,

— prediction,

— updating.

A.2 Splitting stage

The lifting algorithm of the wavelet transform first of all divides the smoothed profile from the jth rung $A_{j,k}$ into "even" and "odd" subsets, in which each sequence contains half as many samples as $A_{j,k}$. The operator is given by:

$$\begin{cases} a_{j+1,k} = A_{j,2k} \\ d_{j+1,k} = A_{j,2k+1} \end{cases} \tag{A.1}$$

where

$A_{0,k} = Z_k$, the original sampled profile.

A.3 Prediction stage

The even and odd subsets are interspersed. If the profile has local correlation structure, the even and odd subsets will be highly correlated. It should thus be possible to predict the odd subset from the even subset with reasonable accuracy.

The prediction stage of the wavelet algorithm consists of predicting the odd subset from the even subset and then removing the predicted value from the odd subset value. The operator is given by:

$$d_{j+1,k} = d_{j+1,k} - \rho\left(a_{j+1,k}\right) \tag{A.2}$$

For the family of interpolating spline wavelets, linear polynomials are used for the prediction. $\rho\left(a_{j+1,k}\right)$ is a weighted prediction of a wavelet coefficient point given by:

$$\rho\left(a_{j+1,k}\right) = \sum_{i=1}^{N} f_i\left(a_{j+1,k}\right) \tag{A.3}$$

The value of $\rho\left(a_{j+1,k}\right)$ is based on the even set, where N denotes how many data points will attend the weighted prediction.

f_i are a set of filtering factors (weighting function) of one wavelet coefficient point, and can be found by employing a “Neville's polynomial interpolation” (see [7][8][9]) with a degree (N–1), with the following recursion:

$$f_i = f_{1,2,\ldots,N}(x) = \frac{(x - x_1) f_{2,\ldots,N}(x) - (x - x_N) f_{1,2,\ldots,N-1}}{(x_N - x_1)} \tag{A.4}$$

Initial coefficients, $f_1, f_2, \ldots, f_N$, are a set of Bezier coefficients of a spline interpolation, with degree (N–1).

For example, if cubic polynomial interpolation is employed to create a weighting function, four neighbouring values will attend a weighted prediction. Five cases should be taken into account:

a) two neighbouring points on either side of an interval;

b) one sample point on the left and three on the right at the left boundary of an interval;

c) vice versa at the right boundary;

d) four sample points on the left; and

e) four sample points on the right.

These cases are considered in order to guarantee boundary “naturalness”, without including any artefacts (all filtering factors are indicated in Table A.1). The result of this is that running-in and running-out lengths of normal filtering techniques are not needed.

Table A.1 — Filter coefficients for cubic polynomial interpolation

Number of samples		$k-7$	$k-5$	$k-3$	$k-1$	$k+1$	$k+3$	$k+5$	$k+7$
on left	on right								
0	4					2,187 5	–2,187 5	1,312 5	–0,312 5
1	3				0,312 5	0,937 5	–0,312 5	0,062 5	
2	2			–0,063	0,562 5	0,562 5	–0,063		
3	1		0,062 5	–0,312 5	0,937 5	0,312 5			
4	0	–0,312 5	1,312 5	–2,187 5	2,187 5				

For example, when there are two samples on the left and two samples on the right, the lifting factors are:

$$f = \left(-\frac{1}{32}, \frac{9}{32}, \frac{9}{32}, -\frac{1}{32}\right) \tag{A.5}$$

and the scalar coefficients can be updated to

$$d_{j+1,k} = d_{j+1,k} - \tfrac{1}{32}\left(-a_{j+1,k-2} + 9a_{j+1,k-1} + 9a_{j+1,k} - a_{j+1,k+1}\right) \tag{A.6}$$

A.4 Update stage

For every level of the multiresolution ladder, the resulting smoother profiles should preserve some of the properties of the original profile, e.g. the same average value and other higher moments. This is achieved in the updating stage.

The updating stage of the wavelet algorithm consists of updating the even subset from the odd subset, in order to preserve as many profile moments as possible. The operator is given by:

$$A_{j+1,k} = a_{j+1,k} + \mu(d_{j+1,k}) \tag{A.7}$$

$\mu(d_{j+1,k})$ is a weighting update given by:

$$\mu(d_{j+1,k}) = \sum_{i=1}^{\tilde{N}} l_i(d_{j+1,k}) \tag{A.8}$$

$\mu(d_{j+1,k})$ is based on the real wavelet coefficients, where $\tilde{N}$ indicates how many wavelet coefficient points will attend the weighting update. The larger $\tilde{N}$ is, the more profile moments are preserved. The l_i are referred to as lifting factors.

The lifting factors can be calculated by the following algorithm. Firstly, an initial moment matrix is defined for all coefficients at the first level of the multiresolution ladder. The moment matrix M is defined by the number of points in the profile s and the value of $\tilde{N}$.

$$M[p,q] = \begin{bmatrix} m_{1,1} & \cdots & m_{1,\tilde{N}} \\ \vdots & m_{p,q} & \vdots \\ m_{s,1} & \cdots & m_{s,\tilde{N}} \end{bmatrix} = \begin{bmatrix} 1 & 1^2 & \cdots & 1^{\tilde{N}} \\ 2 & 2^2 & \cdots & 2^{\tilde{N}} \\ \vdots & \vdots & \ddots & \vdots \\ s & s^2 & \cdots & s^{\tilde{N}} \end{bmatrix} \quad \begin{matrix} 1 \leqslant p \leqslant s \\ 1 \leqslant q \leqslant \tilde{N} \end{matrix} \tag{A.9}$$

Updating the moment matrix requires an indication of how many filtering factors of corresponding wavelet coefficients will contribute to the update. When neighbouring point numbers on each side are the same, the moments can be expressed by:

$$m_{2p,q} = m_{2p,q} + \sum_{t,j} f_i m_{t,q} \tag{A.10}$$

where

$t = 2p - N + 1,\ 2p - N + 3, \cdots, 2p + N - 1$

$i = 1, \cdots, N$

The lifting factors are the solution of the following linear system

$$\begin{bmatrix} m_{2p-\tilde{N}+2,1} & \cdots & m_{2p+\tilde{N},1} \\ \vdots & m_{2p,q} & \vdots \\ m_{2p-\tilde{N}+2,\tilde{N}} & \cdots & m_{2p+\tilde{N},\tilde{N}} \end{bmatrix}_{\tilde{N},\tilde{N}} \begin{bmatrix} l_1 \\ \vdots \\ l_q \\ \vdots \\ l_{\tilde{N}} \end{bmatrix} = \begin{bmatrix} m_{2p+1,1} \\ \vdots \\ m_{2p+1,q} \\ \vdots \\ m_{2p+1,\tilde{N}} \end{bmatrix} \tag{A.11}$$

For example, when a weighting update of a scalar coefficient is considered to be a cubic spline interpolation, the update can be calculated by using four neighbour wavelet coefficients. In this case, the lifting factors are, $l = \left(-\frac{1}{32}, \frac{9}{32}, \frac{9}{32}, -\frac{1}{32}\right)$ and the scalar coefficients can be updated to

$$A_{j+1,k} = a_{j+1,k} + \frac{1}{32}\left(-d_{j+1,k-2} + 9d_{j+1,k-1} + 9d_{j+1,k} - d_{j+1,k+1}\right) \quad \text{(A.12)}$$

A.5 Forward and inverse transforms

To summarize, the forward transform is given as:

$$\text{Split} \begin{cases} a_{j+1,k} = A_{j,2k} \\ d_{j+1,k} = A_{j,2k+1} \end{cases} \quad \text{(A.13)}$$

$$\text{Predict } d_{j+1,k} = d_{j+1,k} - \rho(a_{j+1,k})$$

$$\text{Update } A_{j+1,k} = a_{j+1,k} + \mu(d_{j+1,k})$$

One important property of the lifting scheme is that once the forward transform is defined, the inverse transform can immediately be obtained. The operations are just reversed and the + and – toggled. This leads to the following algorithm for the inverse transform.

$$\text{Update } a_{j+1,k} = A_{j+1,k} - \mu(d_{j+1,k}) \quad \text{(A.14)}$$

$$\text{Predict } d_{j+1,k} = d_{j+1,k} + \rho(a_{j+1,k})$$

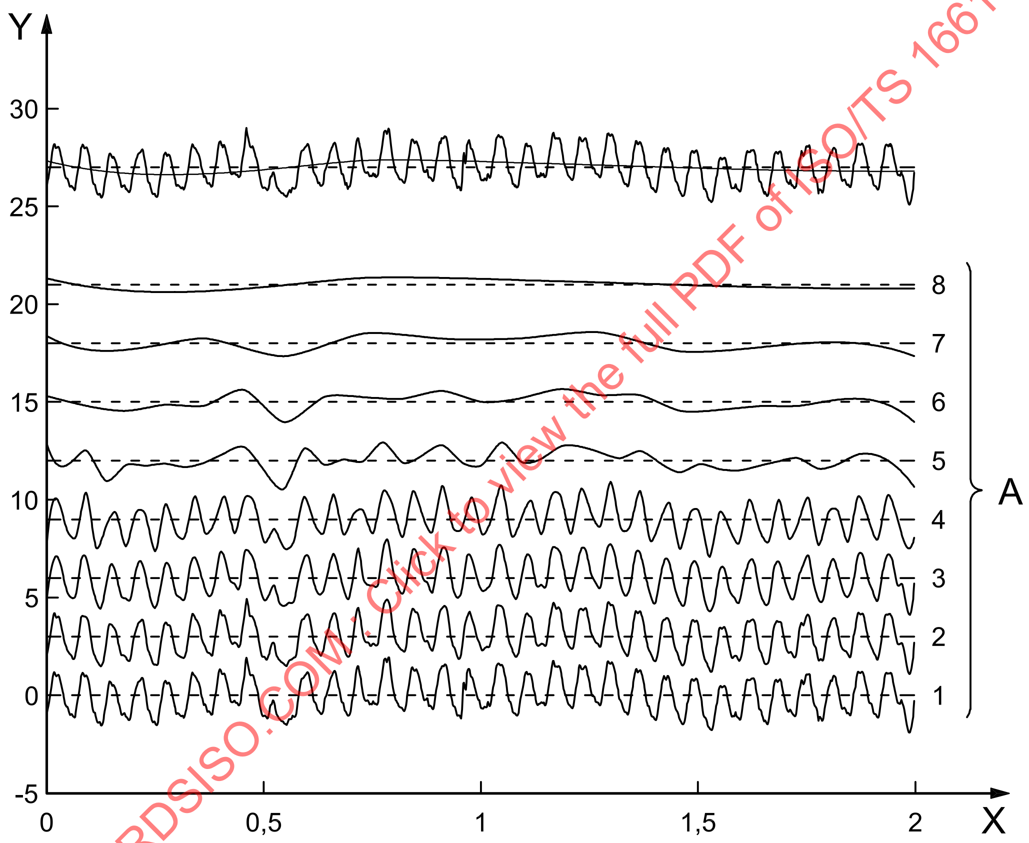
$$\text{Combine } = \begin{cases} A_{j,2k} = a_{j+1,k} \\ A_{j,2k+1} = d_{j+1,k} \end{cases}$$

Annex B
(informative)

Examples of the application of cubic of interpolating spline wavelets

B.1 Profile from a milled surface

The profile is from a milled surface, and is measured with a 5mm tip stylus. Figure B.1 shows the successively “smoothed” profiles, together with the original profile at the top, with the smoothest profile superimposed on top.



Key

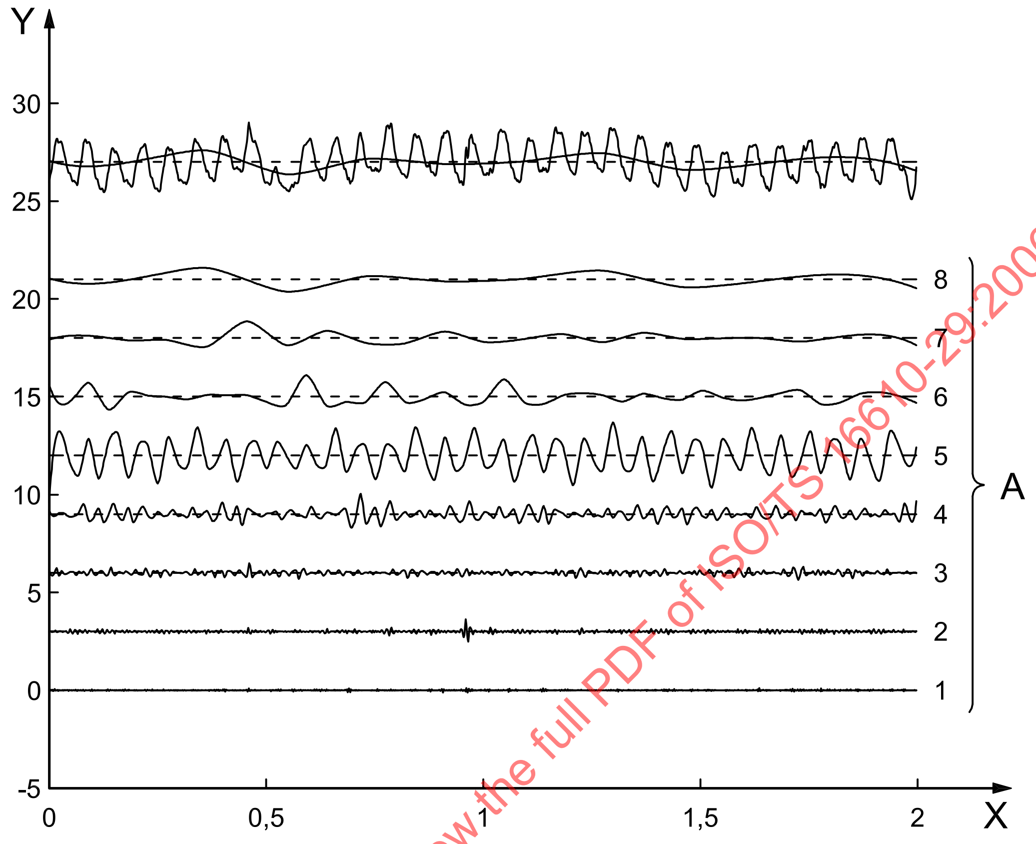
X distance, mm

Y height, µm

A levels

Figure B.1 — Successively smoothed profiles of a milled surface using cubic interpolating spline wavelet

Figure B.2 shows the differences (details) between successive smoothings. The milling marks are easily identifiable at level 5.



Key

X distance, mm

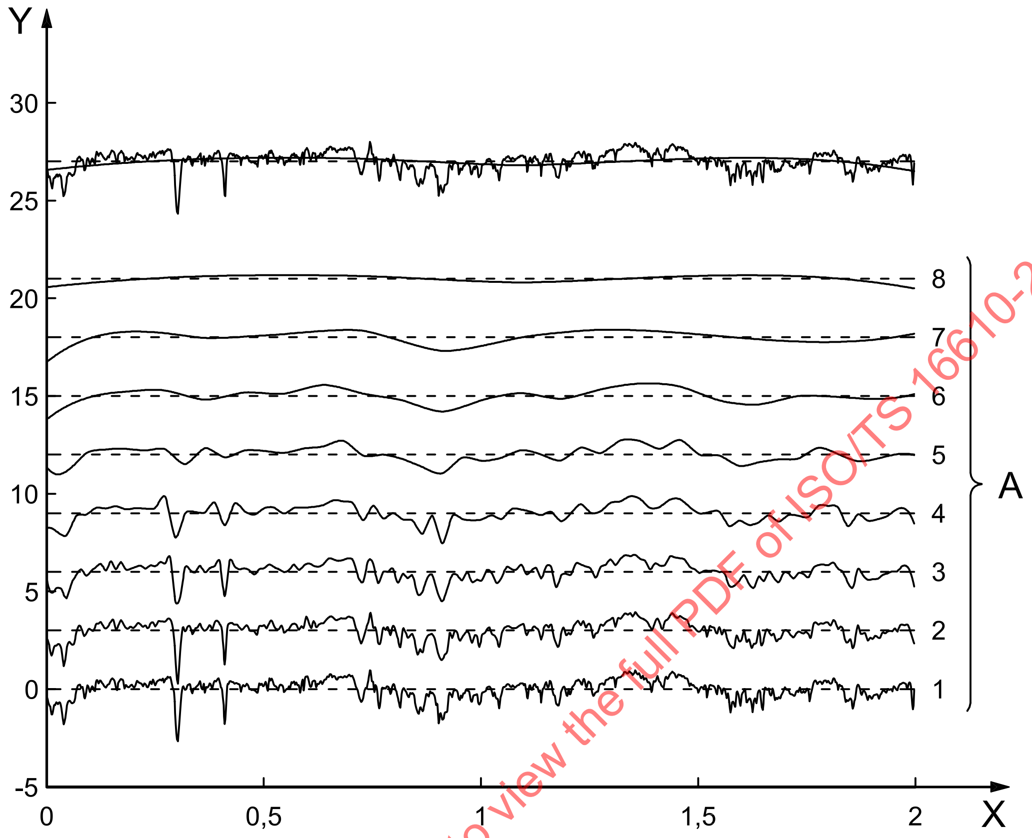
Y height, µm

A levels

Figure B.2 — Differences on a milled surface using cubic interpolating spline wavelet

B.2 Profile from a ceramic surface

The profile is from a rough ceramic surface, and is measured with a 5 mm tip stylus. Figure B.3 shows the successively “smoothed” profiles, together with the original profile at the top.



Key

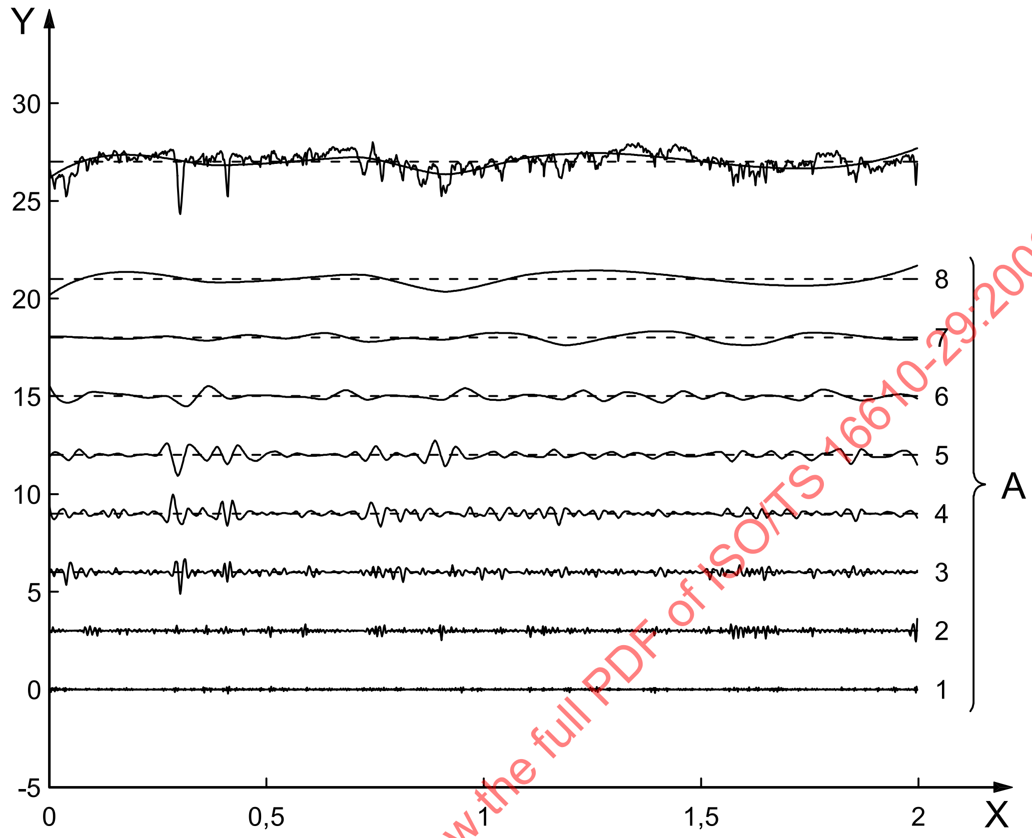
X distance, mm

Y height, µm

A levels

Figure B.3 — Successively smoothed profiles of a ceramic surface using cubic interpolating spline wavelets

Figure B.4 shows the differences (details) between successive smoothings. The deep valleys are easily identifiable at levels 3 and 4, and various asperities can be seen at levels 1 and 2.



Key

X distance, mm

Y height, µm

A levels

Figure B.4 — Differences on a ceramic surface using a cubic interpolating spline wavelet