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Metal Dimensional Change with Temperature

RATIONALE

AIR809A has been reaffirmed to comply with the SAE five-year review policy.

FOREWORD

Changes in this Revision are format/editorial only.

1. SCOPE:

Data are related to the dimensional changes with respect to room temperature and provide accuracy sufficient for use with most aerospace systems. Two basic methods of solution are presented with examples for steady state conditions involving uniform and non-uniform temperature distributions. The answers to the example problems are carried to the fifth place for a comparison of the results between Methods I and II.

1.1 Purpose:

This report is intended to assist in the calculation of metal dimensional changes due to temperature.

2. REFERENCES:

There are no referenced publications specified herein.

3. METHOD I:

This solution uses the mean coefficient of thermal expansion of the material under consideration and the temperature change from room temperature to the final steady state temperature.

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3.1 Uniform Temperature Distribution:

Table 1 lists the mean coefficients of thermal expansion of various materials thru the useful temperature range of the particular material. To solve for the change in dimension of a material with uniform temperature distribution, the following equation is used:

$$\Delta D = (D) (\alpha) (\Delta T) \quad (\text{Eq. 1})$$

where:

ΔD = Change in dimension at a temperature T; (inches)

D = Room temperature dimension of the material; (inches)

α = Mean coefficient of linear thermal expansion for the temperature range from room temperature (70 °F) to a temperature T; (inches/inch - °F)

T = Final steady state temperature; (°F)

ΔT = Temperature change related to room temperature; [(T - 70) °F]

- 3.1.1 The mean coefficient of expansion of a material at a particular temperature may not be given in Table 1. The coefficient is computed by interpolating between the two nearest values.

3.2 Non-Uniform Temperature, Linear & Non-Linear Distribution:

The non-uniform temperature distribution can be either linear or non-linear. The linear temperature gradient requires two known temperatures along dimension D. The temperatures at stations "o" thru "n" are found thru interpolation. The non-linear temperature gradient requires a number of known temperatures along dimension D. A larger number of temperature readings will produce more accurate results. It is quite possible that the known temperatures will not be evenly spaced. It is suggested the temperature vs. dimension D be plotted to find the temperature at any given station. The change in dimension (D) is found by adding the incremental width change of equal width panels, due to the temperature at each station, using the parabolic solution. (Simpson's Rule).

3.2.1 The dimension (D) is divided into an even number of equal width panels.

The equation to compute the change in dimension is as follows:

$$\Delta D = \frac{H}{3} [(Y_o + Y_n) + 4 (Y_1 + Y_3 + Y_5 + \dots + Y_{n-1}) + 2 (Y_2 + Y_4 + Y_6 + \dots + Y_{n-2})] \quad (\text{Eq. 2})$$

where:

- H = Panel width (inches)
 Y_o = Unit growth (in/in) = $(\alpha_o) (\Delta T_o)$; at station 0
 Y_1 = Unit growth (in/in) = $(\alpha_1) (\Delta T_1)$; at station 1
 Y_2 = Unit growth (in/in) = $(\alpha_2) (\Delta T_2)$; at station 2
 Y_n = Unit growth (in/in) = $(\alpha_n) (\Delta T_n)$; at station n
 $\alpha_o \rightarrow n$ = Mean coefficient of expansion at stations "o" through "n"; (in/in- °F)
 $\Delta T_o \rightarrow n$ = Steady state temperature, related to room temperature, at stations "o" through "n"; (°F)
n = Number of panels, must be even.

NOTE: The values given for the mean coefficients of thermal expansion in Table 1 are composites of data received from prime material manufacturers.

3.3 Examples of Method I:

3.3.1 Find the change of a 6.00 inch dimension of stainless steel type 347 material. The final steady state temperature is 1000 °F. From Table 1, the mean coefficient of thermal expansion is 10.1 (10^{-6}) in/in- °F. Using the equation in paragraph 3.1, the change in dimension is as follows:

$$\Delta D = (6.00) (10.1) (10^{-6}) (930) = .05636 \text{ inches} \quad (\text{Eq. 3})$$

The dimension at the final steady state temperature is 6.05636 inches

- 3.3.2 Find the change of a 8.00 inch dimension of stainless steel type 321 material. The final steady state temperature is a linear distribution from 200 °F to 1000 °F. Eight (8) equal panels of 1.00 inch width are arbitrarily chosen. The ΔT at stations "o" through "n" are as follows:

$$\begin{aligned}\Delta T_o &= 130\text{ }^{\circ}\text{F} \\ \Delta T_1 &= 230\text{ }^{\circ}\text{F} \\ \Delta T_2 &= 330\text{ }^{\circ}\text{F} \\ \Delta T_3 &= 430\text{ }^{\circ}\text{F} \\ \Delta T_4 &= 530\text{ }^{\circ}\text{F} \\ \Delta T_5 &= 630\text{ }^{\circ}\text{F} \\ \Delta T_6 &= 730\text{ }^{\circ}\text{F} \\ \Delta T_7 &= 830\text{ }^{\circ}\text{F} \\ \Delta T_8 &= 930\text{ }^{\circ}\text{F}\end{aligned}$$

- 3.3.2.1 From Table 1, the mean coefficients of thermal expansion, at the final steady state temperature for each station, are used for type 321 stainless steel material. The unit growth at stations "o" through "n" are as follows:

$$\begin{aligned}Y_o &= (130) (8.30) (10^{-6}) = 1.079 (10^{-3}) \text{ in/in} \\ Y_1 &= (230) (8.60) (10^{-6}) = 1.978 (10^{-3}) \text{ in/in} \\ Y_2 &= (330) (8.80) (10^{-6}) = 2.904 (10^{-3}) \text{ in/in} \\ Y_3 &= (430) (9.05) (10^{-6}) = 3.891 (10^{-3}) \text{ in/in} \\ Y_4 &= (530) (9.20) (10^{-6}) = 4.876 (10^{-3}) \text{ in/in} \\ Y_5 &= (630) (9.35) (10^{-6}) = 5.890 (10^{-3}) \text{ in/in} \\ Y_6 &= (730) (9.50) (10^{-6}) = 6.935 (10^{-3}) \text{ in/in} \\ Y_7 &= (830) (9.60) (10^{-6}) = 7.968 (10^{-3}) \text{ in/in} \\ Y_8 &= (930) (9.70) (10^{-6}) = 9.021 (10^{-3}) \text{ in/in}\end{aligned}$$

- 3.3.2.2 Using the equation in paragraph 3.2.1, the change in dimension is as follows:

$$\Delta D = \frac{1.00}{3} [(1.079 + 9.021) + 4 (1.978 + 3.891 + 5.890 + 7.968) + 2 (2.904 + 4.876 + 6.935)] (10^{-3})$$

$$\Delta D = .03948 \text{ inches} \quad (\text{Eq. 4})$$

The dimension at the final steady state temperature is 8.03948 inches.

- 3.3.3 Find the change of a 12.00 inch dimension of 52100 low alloy steel material. The final steady state temperature is a non-linear distribution. Ten (10) equal panels of 1.20 inch width are arbitrarily chosen. The temperatures at stations "o" through "n" are shown in Figure 1.

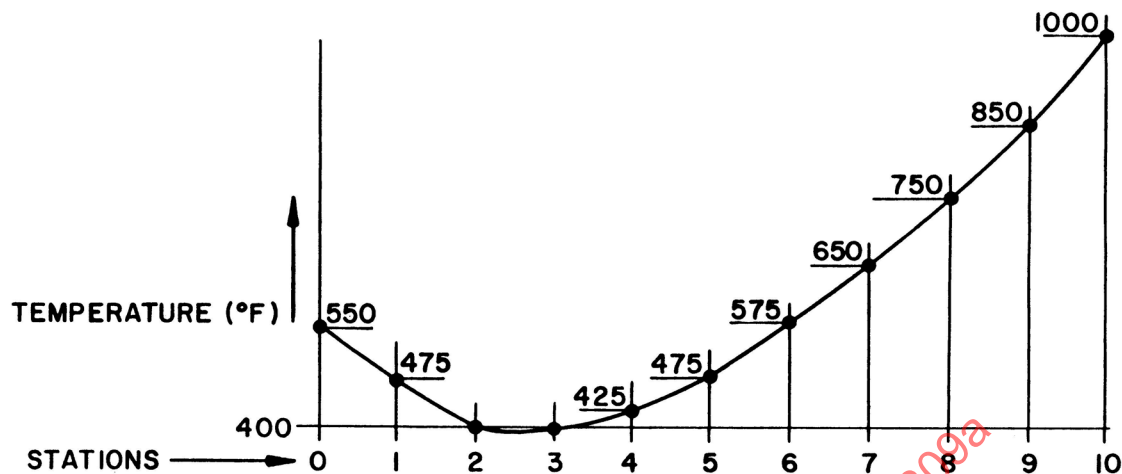


FIGURE 1

3.3.3.1 The mean coefficients of thermal expansion is not given in Table 1 for all of the stations required. Interpolation between the two nearest values is necessary. The value of α at stations "o" through "n" are as follows:

α_0	= 7.335 (10^{-6}) inches/inch - °F
α_1	= 7.2325 (10^{-6}) inches/inch - °F
α_2	= 7.12 (10^{-6}) inches/inch - °F
α_3	= 7.12 (10^{-6}) inches/inch - °F
α_4	= 7.1575 (10^{-6}) inches/inch - °F
α_5	= 7.2325 (10^{-6}) inches/inch - °F
α_6	= 7.3675 (10^{-6}) inches/inch - °F
α_7	= 7.51 (10^{-6}) inches/inch - °F
α_8	= 7.695 (10^{-6}) inches/inch - °F
α_9	= 7.825 (10^{-6}) inches/inch - °F
α_{10}	= 7.95 (10^{-6}) inches/inch - °F

3.3.3.2 The unit growth at stations "o" through "n" are as follows:

Y_0	= (480) (7.335) (10^{-6}) = 3.521 (10^{-3}) in/in
Y_1	= (405) (7.2325) (10^{-6}) = 2.929 (10^{-3}) in/in
Y_2	= (330) (7.12) (10^{-6}) = 2.350 (10^{-3}) in/in
Y_3	= (330) (7.12) (10^{-6}) = 2.350 (10^{-3}) in/in
Y_4	= (355) (7.1575) (10^{-6}) = 2.541 (10^{-3}) in/in
Y_5	= (405) (7.2325) (10^{-6}) = 2.929 (10^{-3}) in/in
Y_6	= (505) (7.3675) (10^{-6}) = 3.720 (10^{-3}) in/in
Y_7	= (580) (7.51) (10^{-6}) = 4.356 (10^{-3}) in/in
Y_8	= (680) (7.695) (10^{-6}) = 5.233 (10^{-3}) in/in
Y_9	= (780) (7.825) (10^{-6}) = 6.103 (10^{-3}) in/in
Y_{10}	= (930) (7.95) (10^{-6}) = 7.393 (10^{-3}) in/in

3.3.3.3 Using the equation in paragraph 3.2.1, the change in dimension is as follows:

$$\Delta D = \frac{1.20}{3} [(3.521 + 7.393) + 4 (2.929 + 2.350 + 2.929 + 4.356 + 6.103) + 2 (2.350 + 2.541 + 3.720 + 5.233)] (10^{-3})$$

$$\Delta D = .04531 \quad (\text{Eq. 5})$$

The dimension at the final steady state temperature is 12.04531.

4. METHOD II:

This solution uses a polynomial equation based on "fitting" the best curve through points of a unit growth versus temperature curve.

4.1 Table 2 lists temperature factors C_0 through C_3 . These factors, used in the polynomial expression, define the unit growth versus temperature curve of a particular material through the useful temperature range.

4.2 Uniform Temperature Distribution:

To solve for the change in dimension of a material with uniform temperature distribution. The following equation is used:

$$\Delta D = (C_0 + C_1 T + C_2 T^2 + C_3 T^3) (10^{-3}) D \quad (\text{Eq. 6})$$

where:

T = Final steady state temperature ($^{\circ}\text{F}$)
 C_0 through C_3 = Temperature factors from Table 2.

4.3 Non-Uniform Temperature, Linear Distribution:

To find the change in dimension of a material subjected to a linear temperature gradient, the following equation is used:

$$\Delta D = \left[C_0 + \frac{C_1 (T_o + T_d)}{2} + C_2 \frac{[(T_o + T_d)^2 - (T_o T_d)]}{3} + C_3 \frac{[(T_o + T_d)^3 - 2 T_o T_d (T_o + T_d)]}{4} \right] (10^{-3}) D \quad (\text{Eq. 7})$$

where:

T_o = Final steady state temperature at $D = \text{zero inches}$ ($^{\circ}\text{F}$)
 T_d = Final steady state temperature at $D = D \text{ inches}$ ($^{\circ}\text{F}$)

4.4 Non-Uniform Temperature, Non-linear Distribution:

To find the change in dimension of a material subjected to a non-linear temperature gradient, the following equation is used to determine the unit growth at stations "o" through "n".

$$Y_o \rightarrow n = [(C_o + C_1 T_o \rightarrow n + C_2 T_o^2 \rightarrow n + C_3 T_o^3 \rightarrow n)] (10^{-3}) \quad (\text{Eq. 8})$$

where:

$Y_o \rightarrow n$ = Unit growth at any station "o" through "n", (in/in).

$T_o \rightarrow n$ = Final steady state temperature at any station "o" through "n", (°F).

The remainder of the solution is as defined in paragraph 3.2.1.

4.5 Examples of Method II:

4.5.1 Find the change in dimension for the example stated in paragraph 3.3.1 where:

$$T = 1000 \text{ } ^\circ\text{F}$$

$$T^2 = 1000 (10^3) \text{ } ^\circ\text{F}$$

$$T^3 = 1000 (10^6) \text{ } ^\circ\text{F}$$

$$C_o = -.63$$

$$C_1 = +8.22 (10^{-3})$$

$$C_2 = +3.29 (10^{-6})$$

$$C_3 = -1.54 (10^{-9})$$

4.5.1.1 Using the equation in paragraph 4.2, the change in dimension is as follows:

$$\Delta D = [-.63 + (8.22) (10^{-3}) (1000) + (3.29) (10^{-6}) (1000) (10^3) - (1.54) (10^{-9}) (1000) (10^6)] (10^{-3}) (6.00)$$

$$\Delta D = .05604 \text{ inches} \quad (\text{Eq. 9})$$

The dimension at the final steady state temperature is 6.05604 inches

4.5.2 Find the change in dimension for the example stated in paragraph 3.3.2.

where:

$$\begin{aligned}
 T_o &= 200 \text{ }^{\circ}\text{F} \\
 T_d &= 1000 \text{ }^{\circ}\text{F} \\
 (T_o + T_d) &= 1.2 (10^3) \text{ }^{\circ}\text{F} \\
 (T_o + T_d)^2 &= 1.44 (10^6) \text{ }^{\circ}\text{F} \\
 (T_o + T_d)^3 &= 1.728 (10^9) \text{ }^{\circ}\text{F} \\
 (T_o T_d) &= .20 (10^6) \text{ }^{\circ}\text{F} \\
 C_o &= -.67 \\
 C_1 &= +8.66 (10^{-3}) \\
 C_2 &= +1.00 (10^{-6}) \\
 C_3 &= 0.00
 \end{aligned}$$

4.5.2.1 Using the equation in paragraph 4.3, the change in dimension is as follows:

$$\Delta D = \left[-.67 + \frac{(8.66) (10^{-3}) (1.2) (10^3)}{2} + \frac{(1.00) (10^{-6}) (1.44 - .20) (10^6)}{3} \right] (10^{-3}) (8.00)$$

$$\Delta D = .03951 \text{ inches} \quad (\text{Eq. 10})$$

The dimension at the final steady state temperature is 8.03951.

4.5.3 Find the change in dimension for the example stated in paragraph 3.3.3.

where:

$$\begin{aligned}
 C_o &= -.42 \\
 C_1 &= +5.76 (10^{-3}) \\
 C_2 &= +3.23 (10^{-6}) \\
 C_3 &= -1.17 (10^{-9})
 \end{aligned}$$

4.5.3.1 Using the equation in paragraph 4.4, the unit growth at stations "o" through "n" are as follows:

$$\begin{aligned}
 T_o &= 550 \text{ }^{\circ}\text{F}; Y_o = 3.530 (10^{-3}) \text{ in/in} \\
 T_1 &= 475 \text{ }^{\circ}\text{F}; Y_1 = 2.919 (10^{-3}) \text{ in/in} \\
 T_2 &= 400 \text{ }^{\circ}\text{F}; Y_2 = 2.326 (10^{-3}) \text{ in/in} \\
 T_3 &= 400 \text{ }^{\circ}\text{F}; Y_3 = 2.326 (10^{-3}) \text{ in/in} \\
 T_4 &= 425 \text{ }^{\circ}\text{F}; Y_4 = 2.522 (10^{-3}) \text{ in/in} \\
 T_5 &= 475 \text{ }^{\circ}\text{F}; Y_5 = 2.919 (10^{-3}) \text{ in/in} \\
 T_6 &= 575 \text{ }^{\circ}\text{F}; Y_6 = 3.738 (10^{-3}) \text{ in/in} \\
 T_7 &= 650 \text{ }^{\circ}\text{F}; Y_7 = 4.367 (10^{-3}) \text{ in/in} \\
 T_8 &= 750 \text{ }^{\circ}\text{F}; Y_8 = 5.223 (10^{-3}) \text{ in/in} \\
 T_9 &= 850 \text{ }^{\circ}\text{F}; Y_9 = 6.091 (10^{-3}) \text{ in/in} \\
 T_{10} &= 1000 \text{ }^{\circ}\text{F}; Y_{10} = 7.400 (10^{-3}) \text{ in/in}
 \end{aligned}$$

4.5.3.2 Using the equation in paragraph 3.2.1, the change in dimension is as follows:

$$\Delta D = \frac{1.20}{3} [(3.530 + 7.400) + 4 (2.919 + 2.326 + 2.919 + 4.367 + 6.091) + 2 (2.326 + 2.522 + 3.738 + 5.223)] 10^{-3}$$

$$\Delta D = .04521 \text{ inches} \quad (\text{Eq. 11})$$

The dimension at final steady state temperature is 12.04521 inches.

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MEAN COEFFICIENT OF LINEAR THERMAL EXPANSION FROM 70 ° F TO 10 ° F (10⁻⁶); INCHES/INCH - ° F

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